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Fracture statistics in the three-dimensional random fuse model

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Abstract. We present numerical simulations for the fracture of the three dimensional random fuse model. The damage accumulated prior to fracture follows a Gaussian distribution, suggesting the absence of long-range correlations. The strength distribution is found to be Lognormal with a logarithmic size effect for the average strength. We relate this result with the distribution of damage clusters.

Key words: Disorder, fracture, random fuse model, size effect, strength distribution.

1. Introduction

The statistical properties of fracture in disordered media are interesting not only in view of practical applications, but also for purely theoretical reasons (Herrmann and Roux, 1990). Microstructural details and the amount of disorder significantly influence the fracture properties and the macroscopic breakdown behavior (Kahng et al., 1988). When the disorder is narrowly (weakly) distributed, materials breakdown without significant precursors. As the disorder increases, substantial damage is accumulated prior to failure and the dynamics resembles percolation. Indeed, in the limit of infinite disorder, the damage accumulation process can exactly be mapped onto a percolation problem (Roux et al., 1988). In the case of broad (strong) disorder, the fracture process is preceded by avalanches of failure events (Hansen and Hammer, 1994; Räisänen et al., 1998; Zapperi et al., 1999) and the crack surface is found to be self-affine.

From a practical applications view point, fracture strength and damage distributions, along with the corresponding size effects have played a significant role in developing probabilistic life design methodologies, 'useful' service life predictive models, and reliability and failure risk analysis models. Traditionally, Weibull and (modified) Gumbel distributions based on 'weakest-link' approach have been widely used to describe the strength of brittle materials. These distributions naturally arise from extreme-value statistics of defect cluster distributions based on the following assumptions (Peterlik and Loidl, 2001): (1) defect clusters are independent of each other, i.e., they do not interact with one another; (2) system failure is governed by the 'weakest-link' hypothesis, and (3) there exists a critical defect cluster size below which the system does not fail, and there exists a relation between the critical size of a defect cluster and the material strength. If the defect cluster size distribution is described by a power-law, then the fracture strength obeys Weibull distribution, whereas an exponential defect cluster size distribution leads to the Gumbel distribution for fracture strengths. However, in heterogeneous materials with broad distribution of disorder, Weibull and Gumbel distributions may not adequately represent the fracture strengths corresponding to the peak load response.

The presence of disorder implies that the mechanical properties of a material (i.e. accumulated damage, fracture energy and strength) should be considered in statistical terms. The application of a standard continuum description based on elastic equations does not capture the effect of fluctuations, which require an explicit description of disorder. A well established way to deal with this problem relies on lattice models, describing the medium as a discrete set of elastic bonds with randomly distributed failure thresholds (Herrmannd and Roux, 1990). In the simplest approximation of a scalar displacement, one recovers the random fuse model (RFM) where a lattice of fuses with random thresholds are subject to an increasing external current. The RFM has been extensively investigated in the literature, mainly by numerical simulations (de Arcangelis et al., 1985, 1989; Sahimi and Goddard, 1986; Kahng et al. 1988; Herrmann and Roux, 1990; Deplace et al., 1996; Zapperi et al., 1999; Nukala and Simunovic, 2003). Most of the results have been obtained in two dimensions, since three dimensional simulations are computationally very expensive as discussed in Batrouni and Hansen (1998) and Räisänen et al. (1998), where a relatively small statistical sampling of different disorder configurations was analyzed.

While the RFM represents a crude approximation of the elastic interactions present in a three dimensional solid, it is still a very useful toy model where one can study the interplay between disorder and elasticity in fracture. In particular, the model takes into account the long-range character of the stress redistribution, but neglects its tensorial structure. Despite this fact, two dimensional simulations have shown a good agreement between the statistical properties of the RFM and those of ceentral force models (Nukala et al., 2005). Here we report an analysis of the statistical properties of fracture strength and accumulated damage using the three dimensional RFM. Although quite interesting, morphological aspects of crack surfaces is beyond the scope of present manuscript and thereby will be considered in a future publication. Our present results on fracture strength and accumulated damage show that the scaling laws derived from two dimensional simulations are followed in three dimensions as well, with only small quantitative variations. We thus conclude that the non-trivial size effects observed in strength and accumulated damage represent a general feature of fracture in disordered media.

2. Model

In the RFM a set of conducting bonds are arranged on a cubic lattice (de Arcangelis et al., 1985). When the local current i_j overcomes a randomly chosen threshold t_j, the fuse burns irreversibly. The thresholds are randomly distributed based on a thresholds probability distribution, $p(t)$. Periodic boundary conditions are imposed in both of the horizontal directions to simulate an infinite system and a constant voltage difference, V, is applied between the top and the bottom plates of the cubic lattice. Numerically, we set a unit voltage difference, $V = 1$, and solve the Kirchhoff equations to determine the current flowing in each of the fuses. Subsequently, for each fuse j, the ratio between the current i_j and the breaking threshold t_j is evaluated, and the bond j_c having the largest value, $\max_j(i_j/t_j)$, is irreversibly removed (burnt). The current is redistributed instantaneously after a fuse is burnt implying that the current relaxation in the lattice system is much faster than the breaking of a fuse. Each time a fuse is burnt, it is necessary to re-calculate the current redistribution in the lattice to determine the subsequent breaking of a bond. The process of breaking of a bond, one at a time, is repeated until the lattice system falls apart. In this work, we assume that the bond breaking thresholds are distributed based on a uniform probability distribution, which is constant between 0 and 1. An alternative would be to study power law distributions with $t \sim r^D$, where r is a uniform random number between [0, 1] and D is the disorder strength exponent, as done in Batrouni and Hansen (1998). Since the robustness of the model behavior with respect to disorder has been extensively studied in the literature we concentrate our effort on a single type of disorder, albeit with extensive statistical sampling and larger lattice system sizes.

Numerical simulation of fracture using large fuse networks is often hampered due to the high computational cost associated with solving a new large set of linear equations every time a new lattice bond is broken. Although the sparse direct solvers presented in Nukala and Simunovic (2003) are superior to iterative solvers in two-dimensional lattice systems, for 3D lattice systems, the memory demands brought about by the amount of fill-in during the sparse Cholesky factorization favor iterative solvers. The authors have developed an algorithm based on a block-circulant preconditioned conjugate gradient (CG) iterative scheme for simulating 3D random fuse networks (Nukala and Simunovic, 2004a). The block-circulant preconditioner was shown to be superior compared with the *optimal* point-circulant preconditioner for simulating 3D random fuse networks (Nukala and Simunovic, 2004a). Since these block-circulant and *optimal* point-circulant preconditioners achieve favorable clustering of eigenvalues, these algorithms significantly reduced the computational time required for solving large lattice systems in comparison with the Fourier accelerated iterative schemes used for modeling lattice breakdown (Batrouni and Hansen (1998) and Ramstad et al. (2004)). Using the algorithm presented in (Nukala and Simunovic, 2004a), we have performed numerical simulations on 3D cube lattice networks. For many lattice system sizes, the number of sample configurations, N_{config}, used are extremely large to reduce the statistical error in the numerical results. In particular, we used $N_{\text{config}} = 50000, 20000, 2512, 1200, 400, 11$ for $L = 10, 16, 24, 32, 48, 64$ respectively (see Table 1).

3. Damage statistics

In the case of two-dimensional RFM models, it has been noted that the final breakdown event is very different from the initial precursors (Nukala et al., 2004). We observed similar behavior of damage accumulation in the 3D RFM models as well. In particular, in the initial stages of damage evolution, the fracture process in the 3D RFM models proceeds through accumulation of damage in a diffusive (uncorrelated) manner similar to that observed in the two-dimensional RFM models. However, as the damage starts to accumulate, the current (stress) enhancements at the crack

Table 1. A summary of the main results of the 3D RFM simulations for uniform thresholds distribution, including the number of configurations used to average the results for each system size. n_p and n_f denote the mean fraction of broken bonds in a lattice system of size L at the peak load and at failure, respectively. Similarly, Δ_p and Δ_f denote the standard deviation of fraction of broken bonds at the peak load and at failure respectively.

L	N_{config}	Cubic			
		n_p	Δ_p	n_f	Δ_f
10	50000	563	57	726	59
16	20000	2108	147	2572	152
24	2512	6692	354	7882	337
32	1200	15329	705	17691	649
48	400	49495	1582	55768	1523
64	11	114243	5704	127040	5378

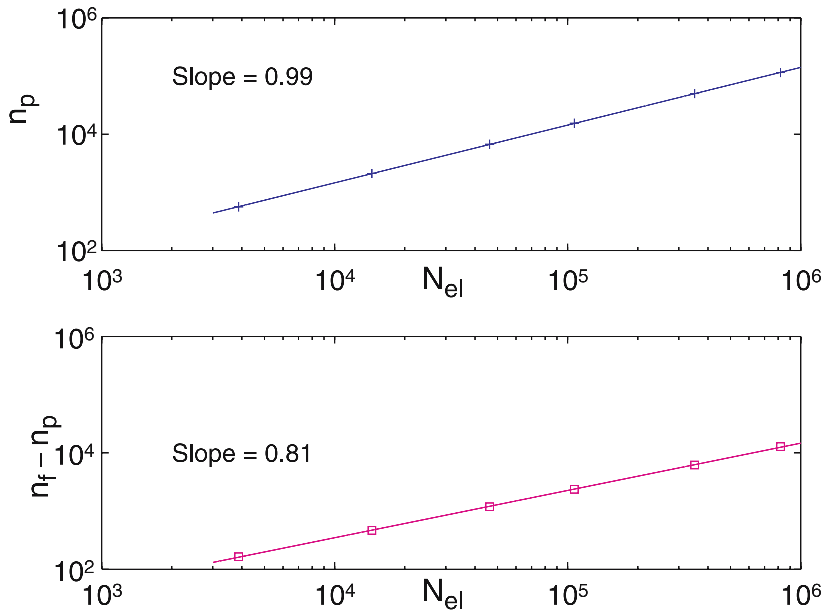

Figure 1. Scaling of number of broken bonds in the pre-peak, and in the post-peak regimes.

tips become relevant, and at around the peak load, which corresponds to the maximum current carried by the network, damage starts to localize yielding a macroscopic crack. Since the initial precursors up to the peak load are different from the final catastrophic event, we analyze here the statistical properties of damage up to the peak load (pre-peak regime), and between the peak load and failure (post-peak regime).

Figure 1 reports the scaling of the average number of broken bonds in the pre-peak (n_p) and in the post-peak ($n_f - n_p$) regimes as a function of the total number of bonds $N_{el} = (3L+2)(L+1)^2$. In the above description, n_p and n_f refer to the average number of broken bonds at the peak load and failure, respectively. The result in Figure 1 shows that n_p increases almost linearly with N_{el}, indicating that the fraction of broken bonds at the peak load is a constant. However, a closer examination of the data presented in Table 1 reveals that this is not strictly correct. By plotting $p_p = n_p/N_{el}$ versus N_{el}, we observed that p_p decreases gradually as N_{el} increases similar to that observed in two dimensional RFM (Nukala et al., 2004). It is difficult, however, to extract a precise law for this trend from the present data. The average number of broken bonds in the post-peak regime, $(n_f - n_p)$, which corresponds to the size of the last catastrophic avalanche, scales as $N_{el}^{0.81}$. In Figure 2 we report the standard deviation in the number of broken bonds at peak load (Δ_p) and failure (Δ_f) as a function of N_{el}. Since there are only a limited number of sample configurations available for the largest system size ($N_{\text{config}} = 11$ for $L = 64$), we used the data up to $L = 48$ to obtain the scaling as $\Delta_p \sim N_{el}^{0.74}$ and $\Delta_f \sim N_{el}^{0.72}$. This implies that the standard deviation in the broken bond density at the peak load, $\Delta_{p_p} = \Delta_p/N_{el}$, is given by $\Delta_{p_p} \sim N_{el}^{-0.26} \sim L^{-0.78}$. Similarly, the standard deviation in the broken bond density at failure, $\Delta_{p_f} = \Delta_f/N_{el}$, is given by $\Delta_{p_f} \sim N_{el}^{-0.28} \sim L^{-0.84}$. It is interesting to compare these exponents of standard deviation in the broken bond density (0.78 and 0.84) with the value 1.20 ± 0.06 obtained in Ramstad et al. (2004) for the power law threshold distribution. This discrepancy may however be due to the differences in the thresholds disorder ditributions (although this is not the case in two dimensions (Nukala et al., 2004)), strong finite size scaling effects associated with small system sizes and lack of extensive statistical sampling for large system sizes.

A more detailed analysis of the damage properties is obtained from the distribution of broken bonds at peak load and failure. The cumulative probability distribution for the damage density at the peak load is defined as the probability $\Pi_p(p_b, L)$ that a system of size L reaches peak load when the fraction of broken bonds equals $p_b = (n_b/N_{el})$, where n_b is the number of broken bonds. A

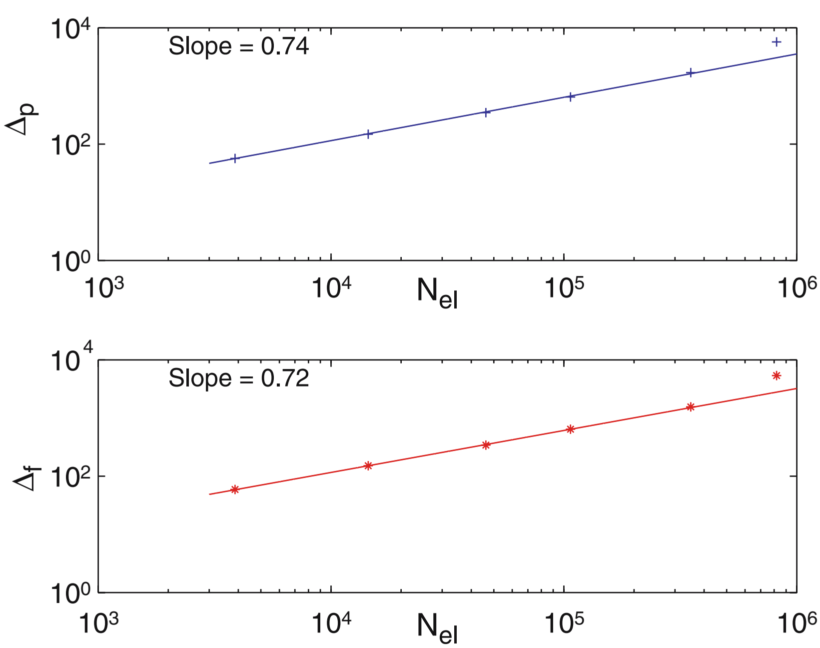

Figure 2. Scaling of standard deviation of the number of broken bonds at the peak load and at failure.

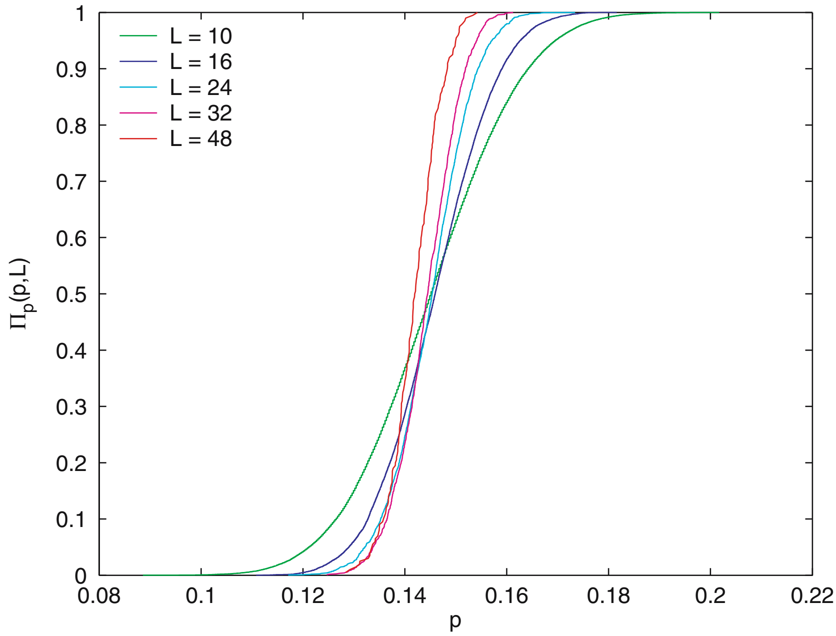

Figure 3. The cumulative probability distribution for the fraction of broken bonds at the peak load for cubic lattices of different system sizes ($L = 10, 16, 24, 32, 48$).

similar cumulative probability distribution $\Pi_f(p_b, L)$ can be defined at failure. Figure 3 presents the cumulative probability distribution for the damage density for various system sizes L. By simply plotting the distribution $\Pi_p(p_b, L)$ in terms of $\bar{p}_p \equiv (n_b - n_p)/\Delta_p = (p_b - p_p)/\Delta_{p_p}$ (and similarly $\Pi_f(p_b, L)$ in terms of $\bar{p}_f \equiv (n_b - n_f)/\Delta_f = (p_b - p_f)/\Delta_{p_f}$), we obtain a very good collapse of the cumulative probability distributions $\Pi_p(p_b, L)$ and $\Pi_f(p_b, L)$ (see Figure 4(a)). The collapse of these damage density distributions for various system sizes at peak load and at failure suggests universality of these cumulative probability distributions in the sense that $\Pi(\bar{p}) = \Pi_p(\bar{p}_p) = \Pi_f(\bar{p}_f)$. The universality of these cumulative probability distributions has been observed in the two-dimensional RFM [18] as well as in the random spring model (Nukala et al., 2005).

Finally, Figure 4(b) indicates that a Gaussian distribution adequately describes the cumulative probability distributions of damage density at the peak load and at failure. Gaussian distribution has been shown to describe these cumulative probability distributions in the two-dimensional RFM model as well. The fact that damage is Gaussian distributed suggests that there is no divergent correlation length at failure consistent with the conclusions of Delaplace et al. (1996) that reported a finite correlation length at failure. Long-range correlations in the damage would imply that the central limit theorem does not hold and hence the normal distribution would not be an adequate fit to the data. The absence of long-range correlation is again in agreement with the hypothesis that fracture is analogous to a first-order transition (Zapperi et al., 1999).

In addition to the damage density distributions, we have also analyzed the distribution of damage clusters at peak load. This quantity is important in the context of the weakest link approach since it determines the type of distribution followed by the

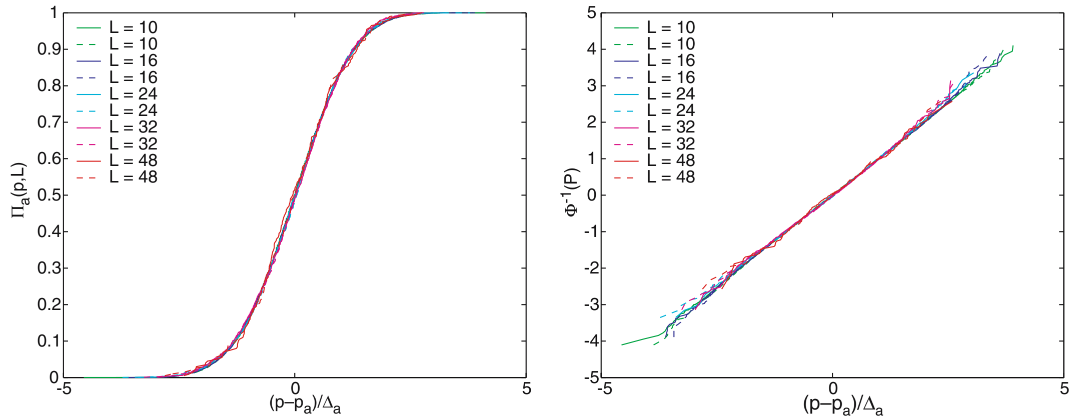

Figure 4. The collapsed cumulative probability distribution for the fraction of broken bonds at the peak load and at failure using cubic lattices of different system sizes ($L = 10, 16, 24, 32, 48$). (a) Cumulative distributions plotted as a function of the reduced variable $\bar{p}_a = (p - p_a)/\Delta_a$ (left). (b) Normal distribution fit in the re-parametrized form (right), where $\Phi(\cdot)$ denotes the standard normal probability function.

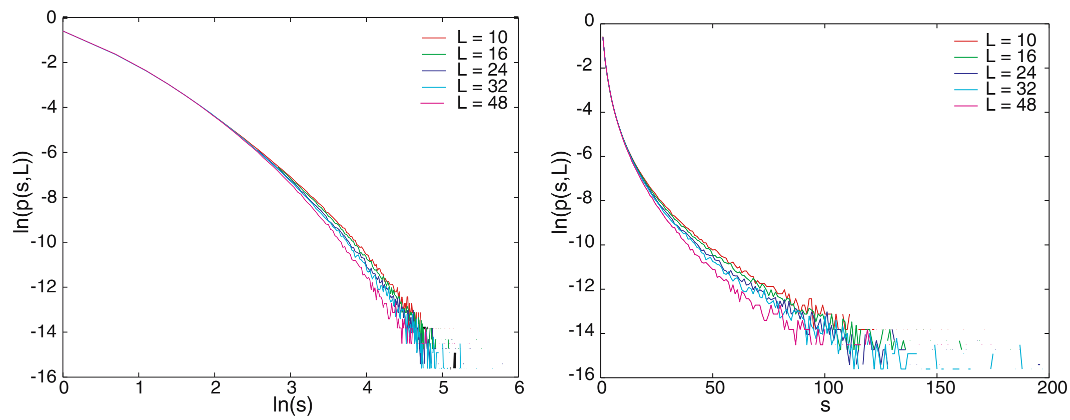

Figure 5. Damage cluster distribution at the peak load in cubic lattices of system sizes $L = \{10, 16, 24, 32, 48\}$. (a) Power-law fit: $p(s, L) \sim \beta\ s^{-\beta-1}$. (b) Exponential fit: $p(s, L) \sim \beta\ \exp(-\beta\ s)$. The cluster distribution data for different lattice system sizes does not collapse on to a single straight line as it should, if the data were to follow either a power-law or an exponential distribution.

strength (Hermann and Roux, 1990). In particular, a power law defect cluster size distribution leads to a Weibull fracture strength distribution, whereas an exponential defect cluster size distribution leads to a Gumbel type fracture strength distribution. However, the results presented in Figure 5 indicate that the distribution is neither power law type nor exponential type. As it will be shown in the next section, the strength distribution is neither Weibull nor Gumbel type, but is a Lognormal distribution. Along with the analysis presented in the following section, these cluster distribution results provide an explanation for the inadequacy of Weibull and (modified) Gumbel distributions to represent the fracture strength distribution of broadly disordered materials.

We have also considered the distribution of the largest cluster at the peak load and found that it follows a Lognormal distribution as well. Figure 6 presents the cumulative probability distribution of largest cluster sizes at the peak load for various

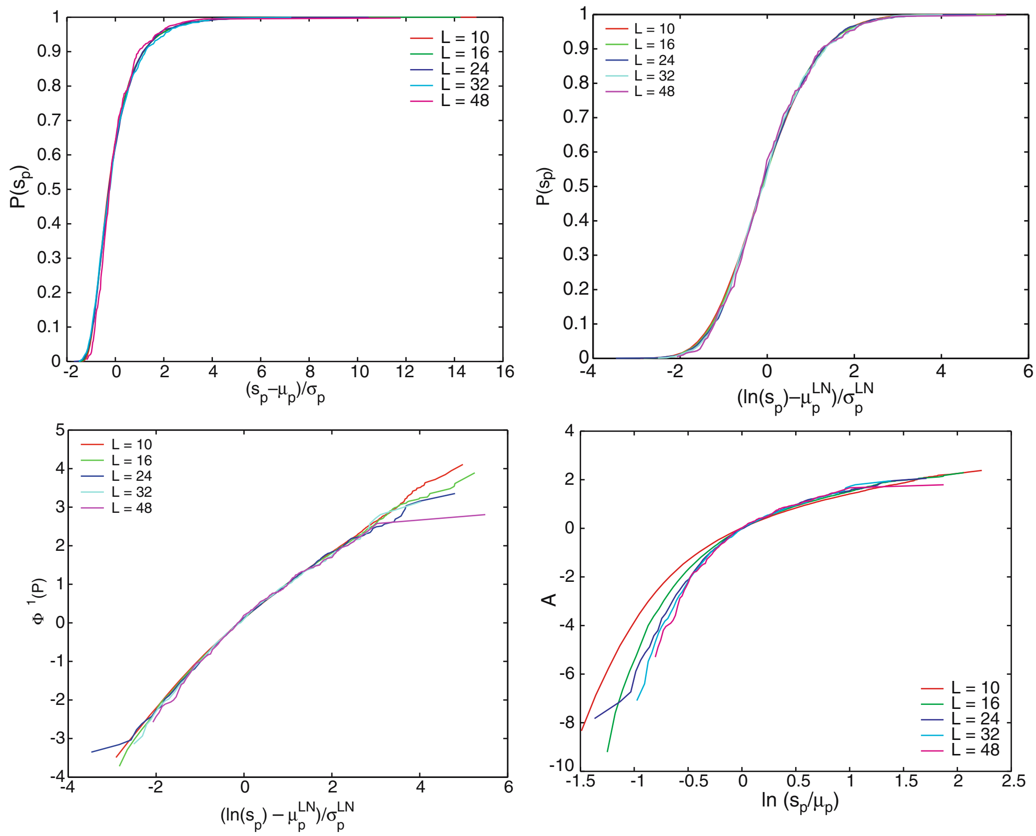

Figure 6. Cumulative probability distribution for largest defect cluster size at the peak load in cubic lattices of system sizes $L = \{10, 16, 24, 32, 48\}$. (a) Data plotted as a function of normal coordinate $\bar{s}$. However, the distribution is clearly not normal (top left). (b) Lognormal distribution fit (top right). (c) Re-parametrized form of Lognormal fit. μ_p^{LN} and σ_p^{LN} refer to the mean and the standard deviation of the logarithm of s_p (bottom left). (d) Re-parametrized form of Weibull fit (bottom right), where $A = \ln(-\ln(1-P))$.

system sizes. Although a reasonable collapse of the distributions is obtained when a normal coordinate $\bar{s} \equiv (s_p - \mu_p)/\sigma_p$ is used in Figure 6(a), the distribution is clearly not normal. On the other hand, a Lognormal distribution as shown in Figure 6(b) provides an excellent fit for the largest cluster distribution at the peak load. This is evident in Figure 6(c) also, where a re-parametrized form of Lognormal distribution is presented. An excellent collapse of the data for different system sizes can be clearly seen. However, a minute deviation from the straight line behavior can also be seen in this re-parametrized Lognormal plot. In comparison, a Weibull fit as shown in Figure 6(d) (a reparametrized plot) is clearly inadequate to represent the largest cluster size distribution data at the peak load.

4. Fracture strength distribution and size effects

Conventionally, Weibull and Gumbel distributions are used to fit the fracture strength data for brittle materials (Jayatilaka and Trustrum, 1977; Orlovskaja et al., 1997). However, as Weibull mentioned in his pioneering paper (Weibull, 1951), the Weibull distribution should be considered as an empirical one on an equal footing with other type of distributions (Orlovskaja et al., 1977). In material science applications, Log-normal, power law, Gamma, Type-I extreme value, and bimodal distributions are also often used for describing the fracture strength distribution (Sigl, 1992; Lissart and Lamon, 1997; Orlovskaja et al., 1997; Peterlik and Loidl, 2001). This is particularly the case when the disorder is broadly distributed such that failure is not governed by the 'weakest-link' hypothesis.

In the 'weakest-link' hypothesis, the fracture strength is determined by the presence of few critical defect clusters, and is defined as the stress required for breaking the very first 'weakest-link' in the system. In materials with broad disorder, the breaking of the very first bond ('weakest-link') does not usually lead to the entire system failure (as is manifested with the scaling $n_p \sim N_{el}^b$ with $b = 0.93$ for 2D RFM (Nukala et al., 2004) and $n_p \sim N_{el}^b$ with $b = 0.99$ for 3D RFM (see Figure 1)), and hence a fracture strength distribution based on the importance of very first bond failure may not be applicable. In addition, since the subsequent bond failure is controlled not only by 'weakest' bonds with smallest thresholds but also by the stress concentration and shielding effects around the crack clusters, the crack cluster size distribution in the RFM at the peak load develops its own form, quite different from the initial defect cluster size distribution in randomly diluted networks. In general at the peak load of RFM simulations, the crack cluster distribution follows a generalized Gamma-distribution, and in this case, neither the Weibull nor the Gumbel distributions fit the fracture strength distribution accurately. In the 3D RFM models, the crack cluster distribution is represented neither by a power law nor by an exponential distribution (see Figure 5). Similar results have been obtained even in 2D RFM simulations (Nukala et al., unpublished).

The results of the simulations allow to determine the form of the fracture strength distribution and its dependence on the lattice size. Figures 7(a) and 7(b) present the modified Gumbel and Weibull fits for the fracture strength distribution of the cubic random fuse model:

$$A = k\left(\frac{1}{\sigma_f^{\delta}}\right) - \ln c \tag{1}$$

for the modified Gumbel distribution, and

$$A = m \ln\left(\frac{1}{\sigma_f}\right) - \ln c \tag{2}$$

for the Weibull distribution. In Eqs. (1) and (2), k, δ, c and m are constants, and A is defined as

$$A = -\ln\left[-\frac{\ln\left(1 - P(\sigma_f)\right)}{L^d}\right] \tag{3}$$

where $P(\sigma_f)$ denotes the cumulative distribution, σ_f denotes the fracture strength defined as the stress corresponding to the peak load of the lattice system response, and $d = 2$ in 2D and $d = 3$ in 3D. From these figures, it is clear that fracture strength data for different lattice system sizes does not collapse on to a single straight line as it should, if the data were to follow Eq. (1) or (2). This indicates that neither

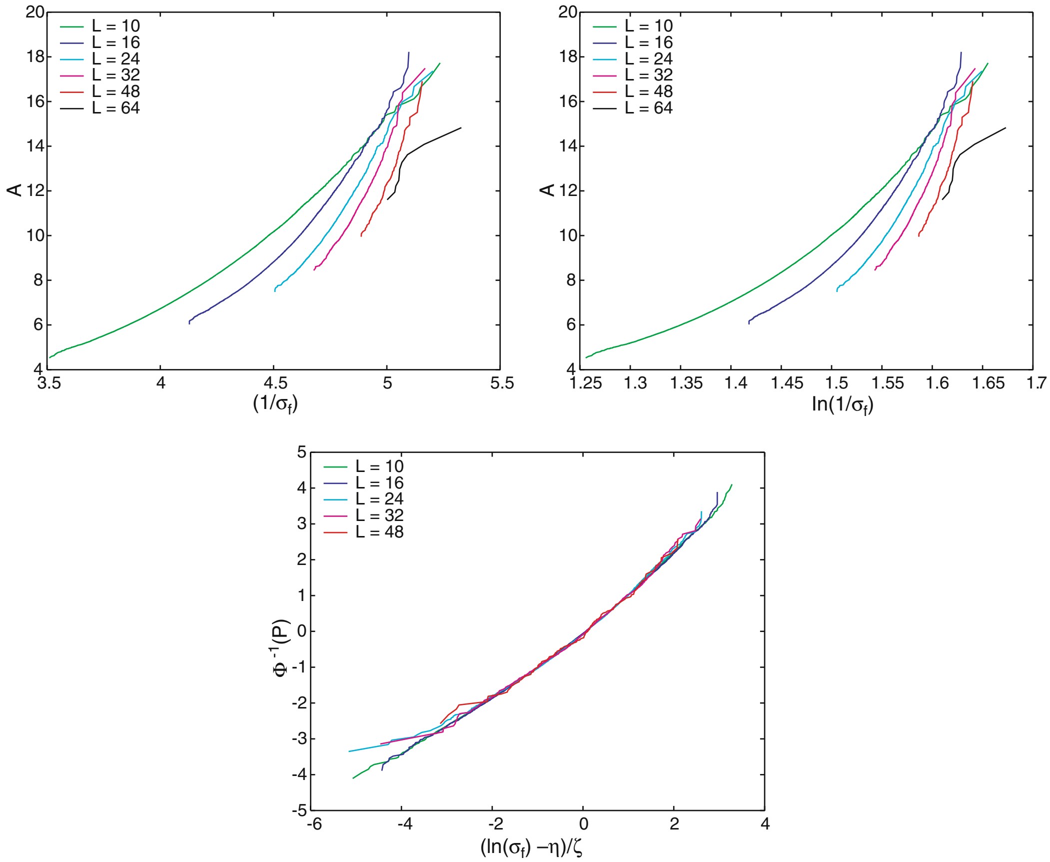

Figure 7. Probability distribution fits for fracture strengths at the peak load in a cubic lattice network for different lattice system sizes $L = \{10, 16, 24, 32, 48, 64\}$. (a) Modified Gumbel distribution (top left). (b) Weibull distribution (top right). (c) Lognormal distribution fit (bottom). Since the data for different lattice system sizes does not collapse onto a single curve, modified Gumbel and Weibull distributions may not be adequate fits for representing the fracture strengths of a cubic lattice with random thresholds. On the other hand, the collapse of the data in the reparametrized Lognormal distribution fit suggests that the Lognormal distribution describes the fracture strength distribution adequately.

modified Gumbel nor Weibull distributions may represent the fracture strengths distribution accurately in three dimensions. Similar conclusion has been drawn for two dimensional random fuse (Nukala and Simunovic, 2004) and random spring models (Nukala et al., 2005). In Figure 7(c), we test the Lognormal description for fracture strengths by plotting the inverse of the cumulative probability, $\Phi^{-1}(P(\sigma_f))$, against the standard Lognormal variable, ξ. In the above description, $\Phi(\cdot)$ denotes the standard normal probability function, and $\xi = (\ln(\sigma_f) - \eta)/\zeta$, where η and ζ refer to the mean and the standard deviation of the logarithm of σ_f. Figure 7(c) clearly indicates that the fracture strength distribution obtained for different lattice system sizes collapses onto a single curve, albeit minute deviation from straight line behavior is evident. In Nukala and Simunovic (2004) and Nukala et al. (2005), similar collapse of the data onto Lognormal distribution has been obtained for two dimensional random fuse (Nukala and Simunovic, 2004) and random spring models (Nukala et al., 2005), wherein different lattice system sizes, different lattice topologies (triangular

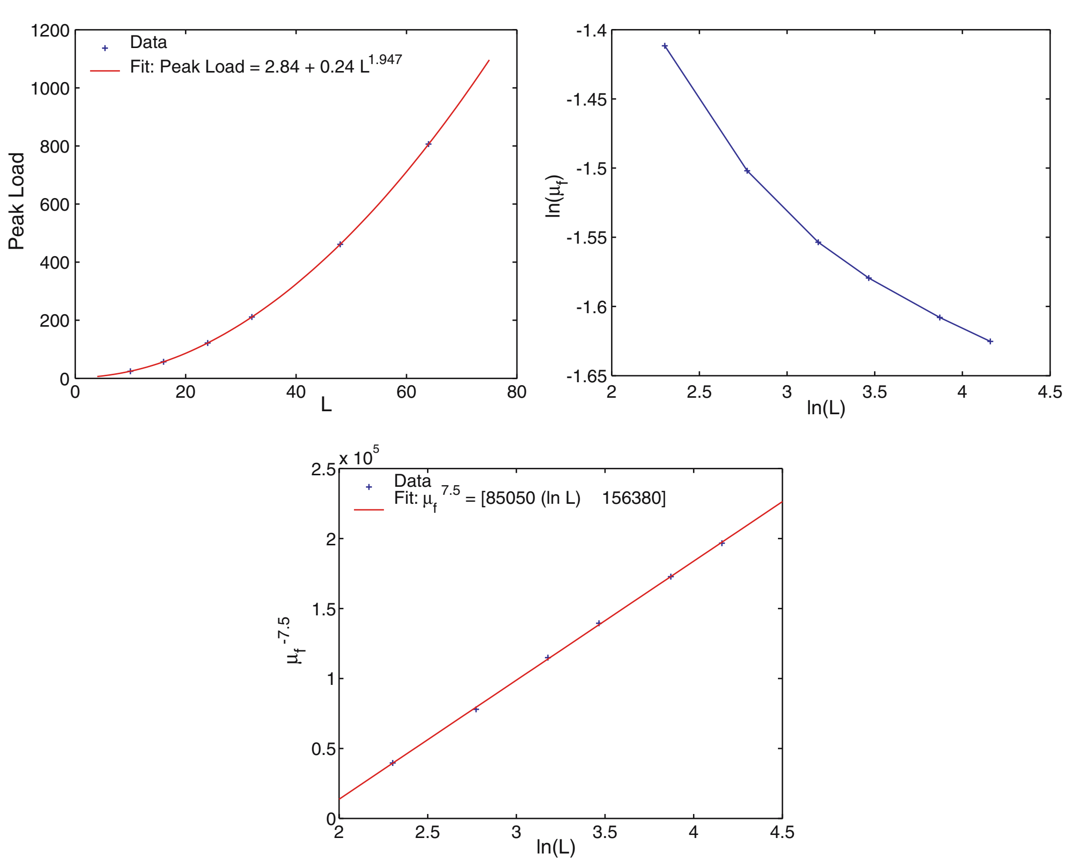

Figure 8. Scaling law for the mean fracture strength (a) Proposed scaling law for the average peak load (top left). (b) Weibull fit for the mean fracture strength (top right). (c) Modified Gumbel fit based on the scaling law: $\mu_f^{\delta} = 1/(A_1 + B_1 \ln L)$ (bottom).

and diamond), and different threshold distributions have been considered. Such an agreement between 3D and 2D RFM and random spring models suggests the notion of a universality of fracture strength distribution in broadly disordered materials. That is, $P(\sigma \leqslant \sigma_f) = \Psi(\xi)$, where Ψ is a universal scaling function such that $0 \leqslant \Psi \leqslant 1$.

A theoretical analysis for the adequacy of Lognormal distribution to represent the fracture strength distribution in broadly disordered materials is presented in Nukala and Simunovic (2004). Mathematically, the Lognormal distribution can be understood to have evolved as a consequence of multiplicative nature of large number of random distributions representing the stress scale factors necessary to break the subsequent 'primary' bonds (by definition, an increase in applied stress is required to break a 'primary' bond) leading up to the peak load (Nukala and Simunovic, 2004). As long as the number of 'primary' broken bonds is large, the fracture strength probability distribution approaches a Lognormal distribution (by central limit theorem) irrespective of the precise character of the individual stress scale factor random distributions (Nukala and Simunovic, 2004). We have also used the normal distribution to collapse the fracture strength data. Although the data collapse is reasonable, the results of Kolmogorov–Smirnov goodness-of-fit test indicated that normal distribution is not as good as that of Lognormal distribution.

Table 2. Peak load.

L	N_{config}	3D Cubic Lattice	
		Mean	SD
10	50000	24.38	1.16
16	20000	57.01	1.62
24	2512	121.82	2.25
32	1200	211.04	2.98
48	400	461.5	4.72
64	11	806.4	14.4

The mean fracture strength data for different lattice system sizes is reported in Table 2. It has been shown in Nukala and Simunovic (2004) that for the two-dimensional RFM the peak load scales as $\bar{I}_{\text{peak}} = C_0 L^{\alpha} + C_1$, where C_0 and C_1 are constants. Correspondingly, the mean fracture strength defined as $\mu_f = \frac{\bar{I}_{\text{peak}}}{L^{d-1}}$, is given by $\mu_f = C_0\ L^{\alpha-d+1} + \frac{C_1}{L^{(d-1)}}$, where $d = 2$ in 2D and $d = 3$ in 3D. We have used the same scaling law in the two-dimensional random thresholds spring model as well, obtaining a good agreement. The results for three dimensional fuse models are presented in Figure 8(a), where it is shown that a similar scaling law holds with $\alpha = 1.95$. In the Figure 8(b), deviation from the straight line behavior indicates that a power law fit $\mu_f \sim L^{-\frac{2}{m}}$ consistent with a Weibull distribution does not provide an adequate fit to the data. We have also verified the scaling law of form $\mu_f^{\delta} = 1/(A_1 + B_1 \ln L)$ that is consistent with a modified Gumbel distribution for fracture strengths (Duxbury et al., 1986, 1987; Beale and Duxbury, 1988; Beale and Srolovitz, 1988). The results presented in Figure 8(c) indicate that the scaling law $\mu_f^{\delta} = 1/(A_1 + B_1 \ln L)$ may represent the mean fracture strengths reasonably well even though the Gumbel distribution is inadequate to represent the cumulative fracture strength distribution. However, the very high value of $\delta = -7.5$ for the exponent and the high values for the coefficients A_1 and B_1 (shown in the inset of Figure 8(c)) indicate the divergence behavior as the lattice system approaches failure (Beale and Duxburg, 1988; Beale and Srolovitz, 1988).

Since a very small negative exponent $(\alpha - d + 1)$ is equivalent to a logarithmic correction, i.e., for $(d - 1 - \alpha) \ll 1$, $L^{\alpha-d+1} \sim (\ln L)^{-\psi}$, an alternative expression for the mean fracture strength may be obtained as $\mu_f = \mu_f^{\star}/(\ln L)^{\psi} + c/L^{(d-1)}$, where $\mu_f^{\star}$ and c are constants that are related to the constants C_0 and C_1. This suggests that the mean fracture strength of the lattice system decreases very slowly with increasing lattice system size, and scales as $\mu_f \approx \frac{1}{(ln\ L)^{\psi}}$, with $\psi \approx 0.15$, for very large lattice systems.

5. Conclusions

In this paper we have reported the result of numerical simulations of the RFM in three dimensions. This model represents probably the simplest model of fracture in a disordered medium and its results have been shown to be equivalent to more complicated central force models, at least in two dimensions (Nukala et al., 2005). Here, we addressed the problem of fluctuations in the accumulated damage and in the strength. According to the weakest link argument, the strength distribution and the related size effect should depend on the nature of the defect distribution. The argument is, however, not necessarily valid when a considerable amount of damage is accumulated prior to failure as in the case we treated. In fact, we find that the strength distribution is Lognormal, with a logarithmic size effect. A similar result was also found in the two-dimensional RFM and spring network, indicating that such a size effect is probably a generic feature in the fracture of broadly disordered media.

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