Information Arbitrage and Optimal Execution

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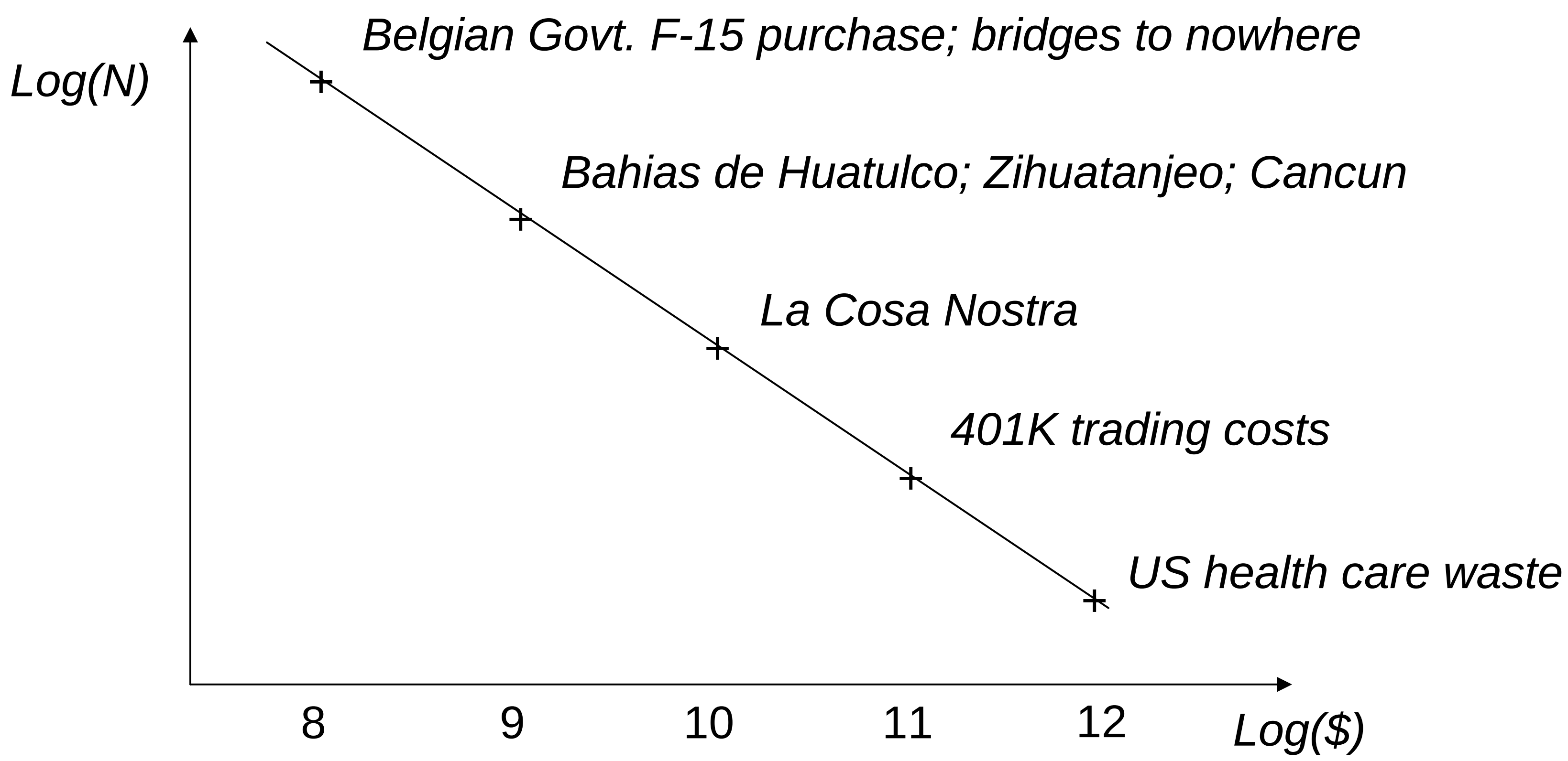
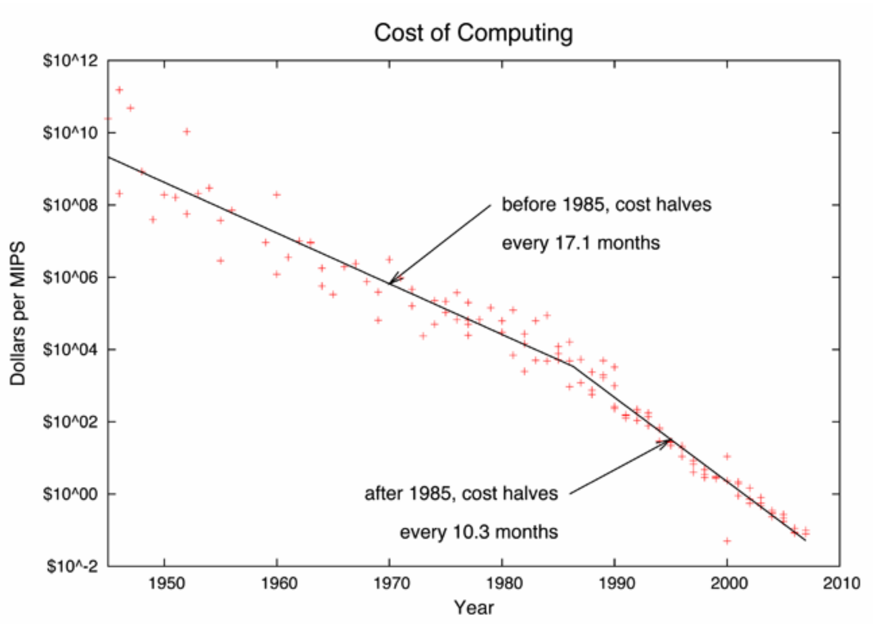
Corruption as Self-organized, Collective Phenomena

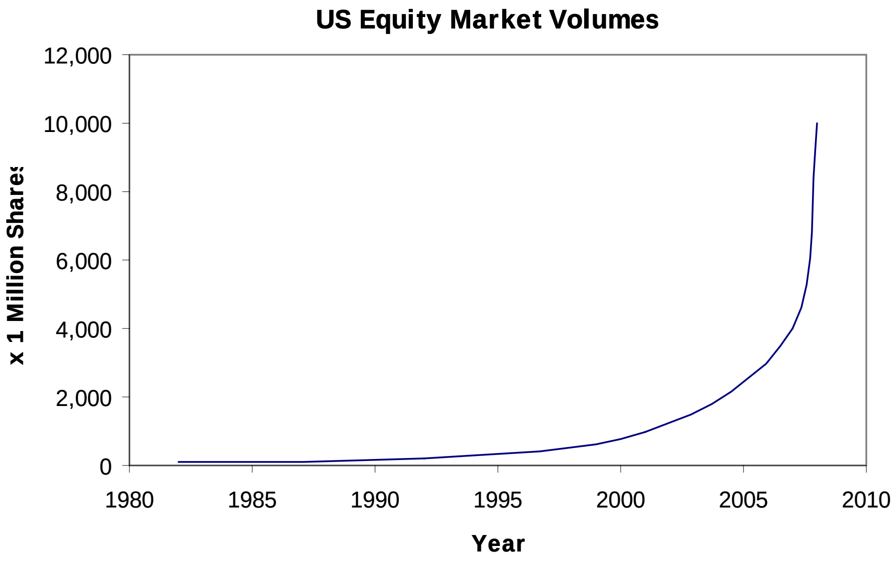
Frequency of corruption scandal types by cost to society



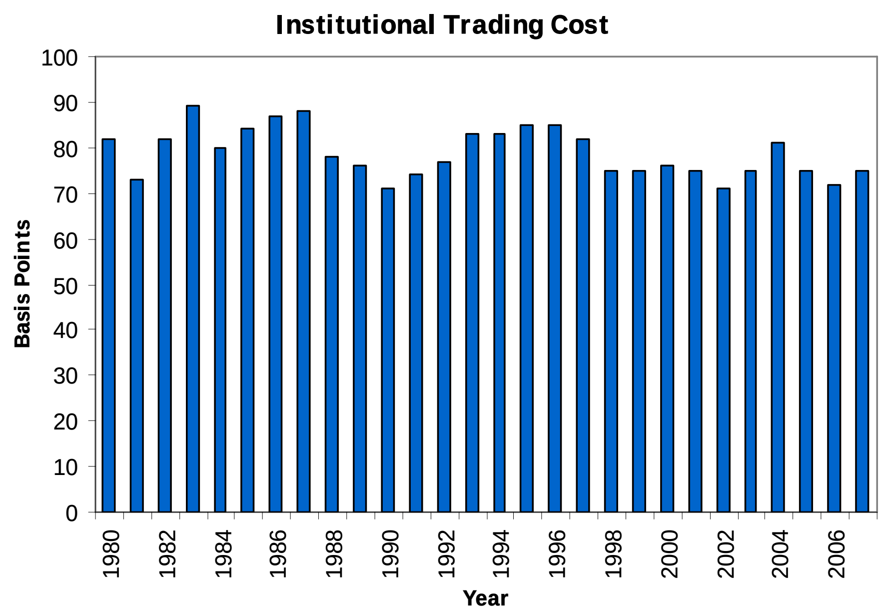
Trends



Cost of Computing
before 1985, cost halves every 17.1 months
after 1985, cost halves every 10.3 months
Dollars per MIPS
Year



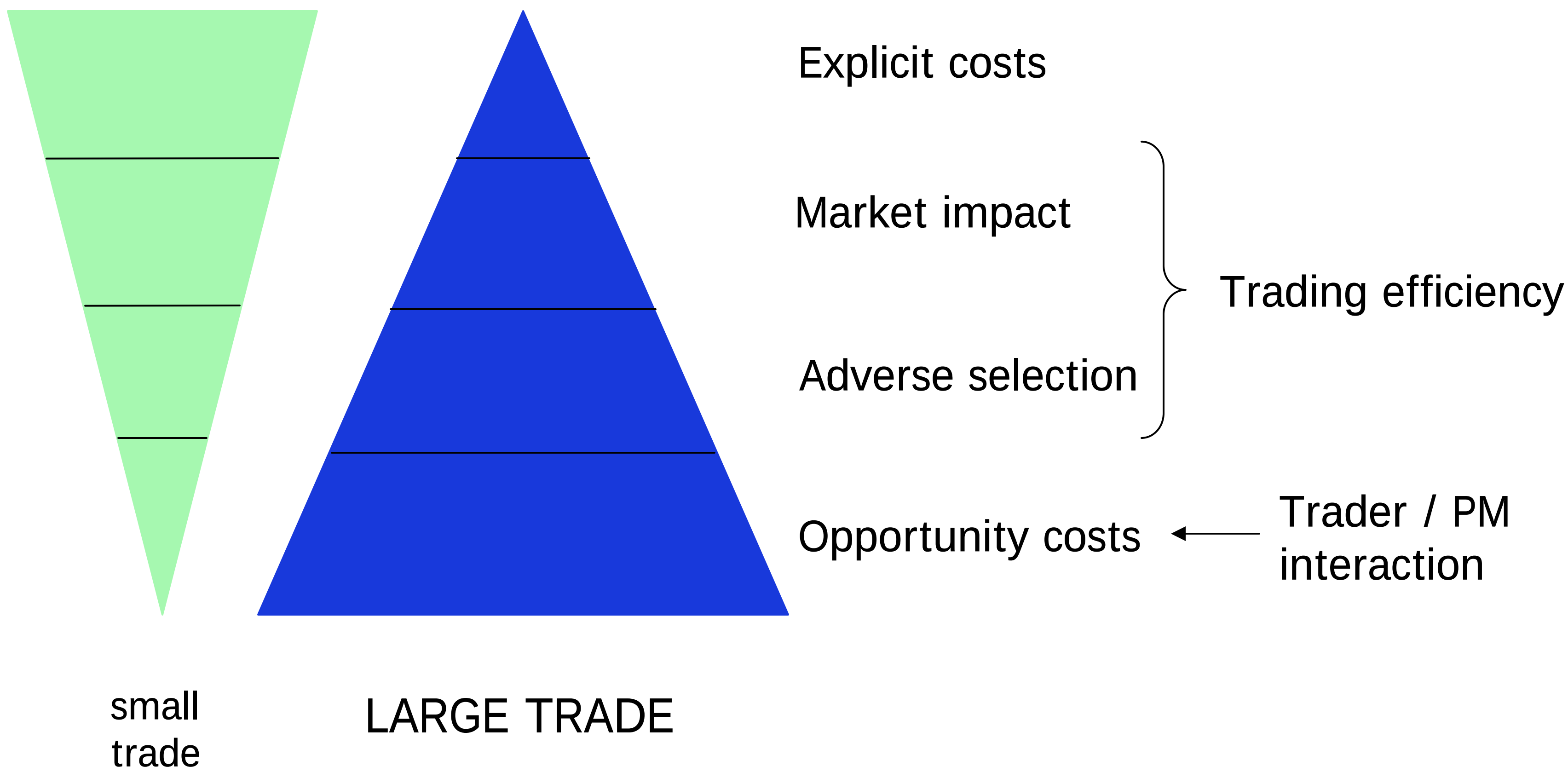
US Equity Market Volumes
x 1 Million Shares
Year



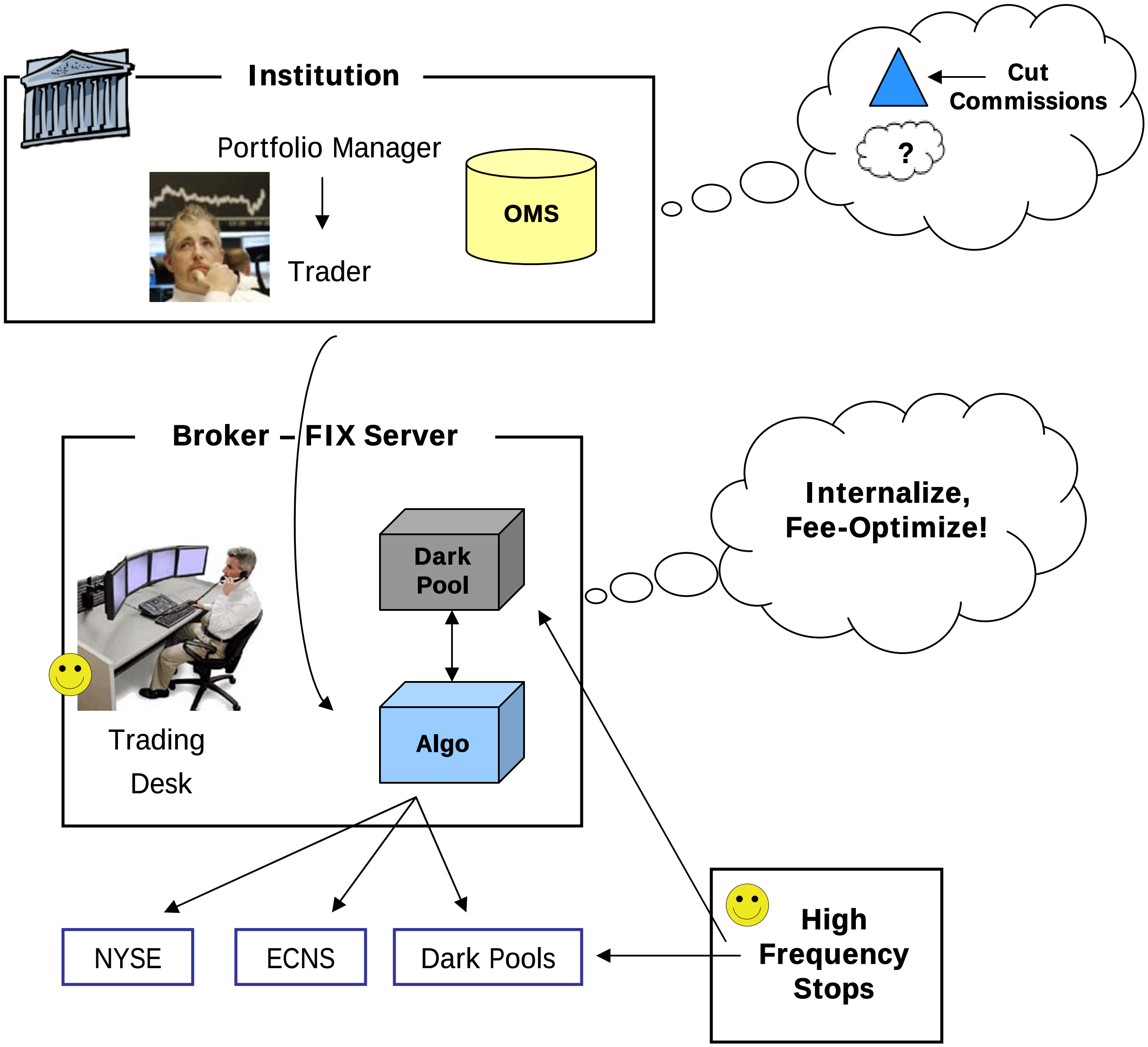
Institutional Trading Cost
Basis Points
Year

The Cost Equation

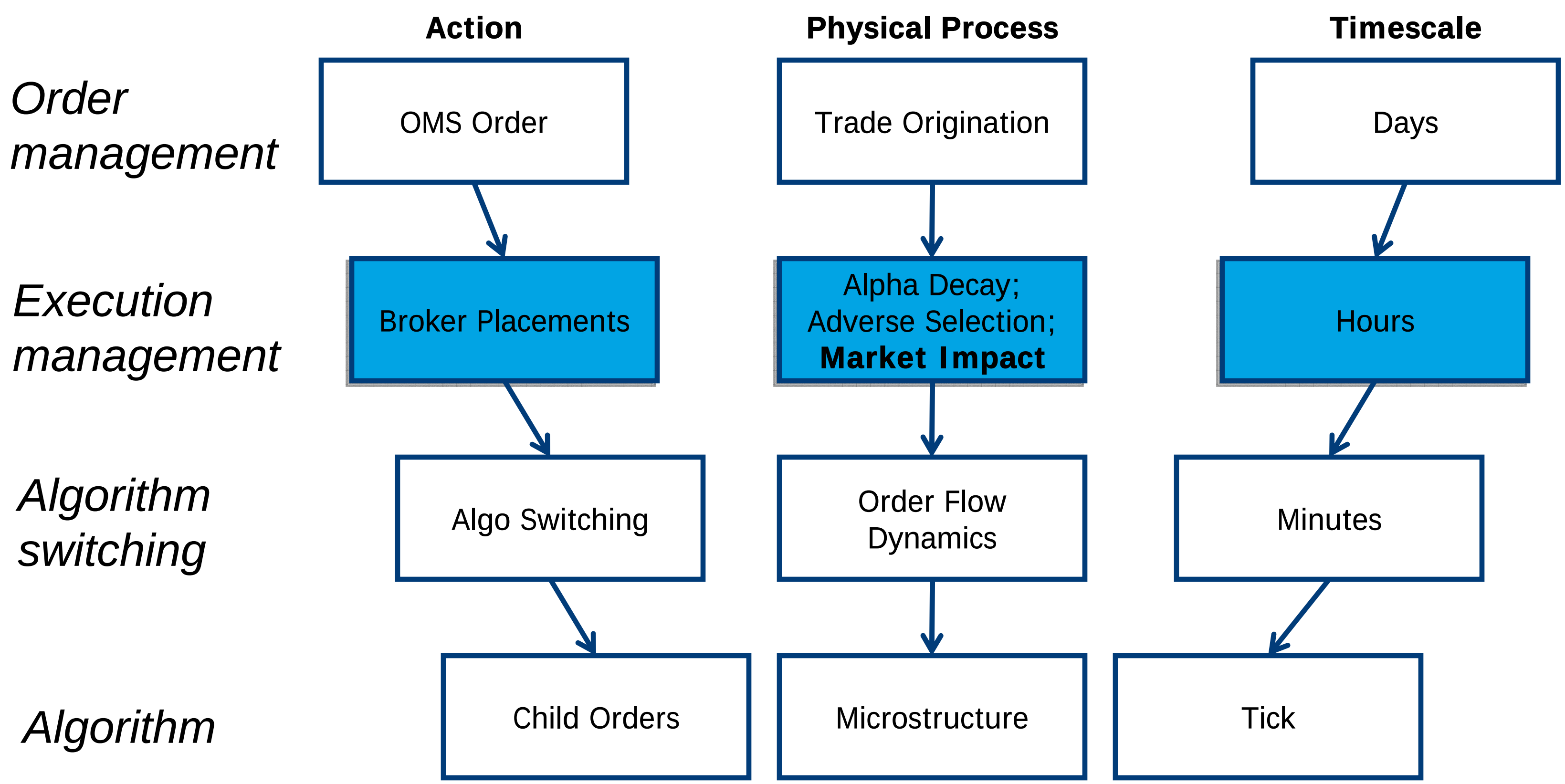
Aggregate 401K shortfall costs (pension funds): $\$10^{11}/yr \approx \frac{1}{2}\Delta(GDP)$



The Economics of Order Flow



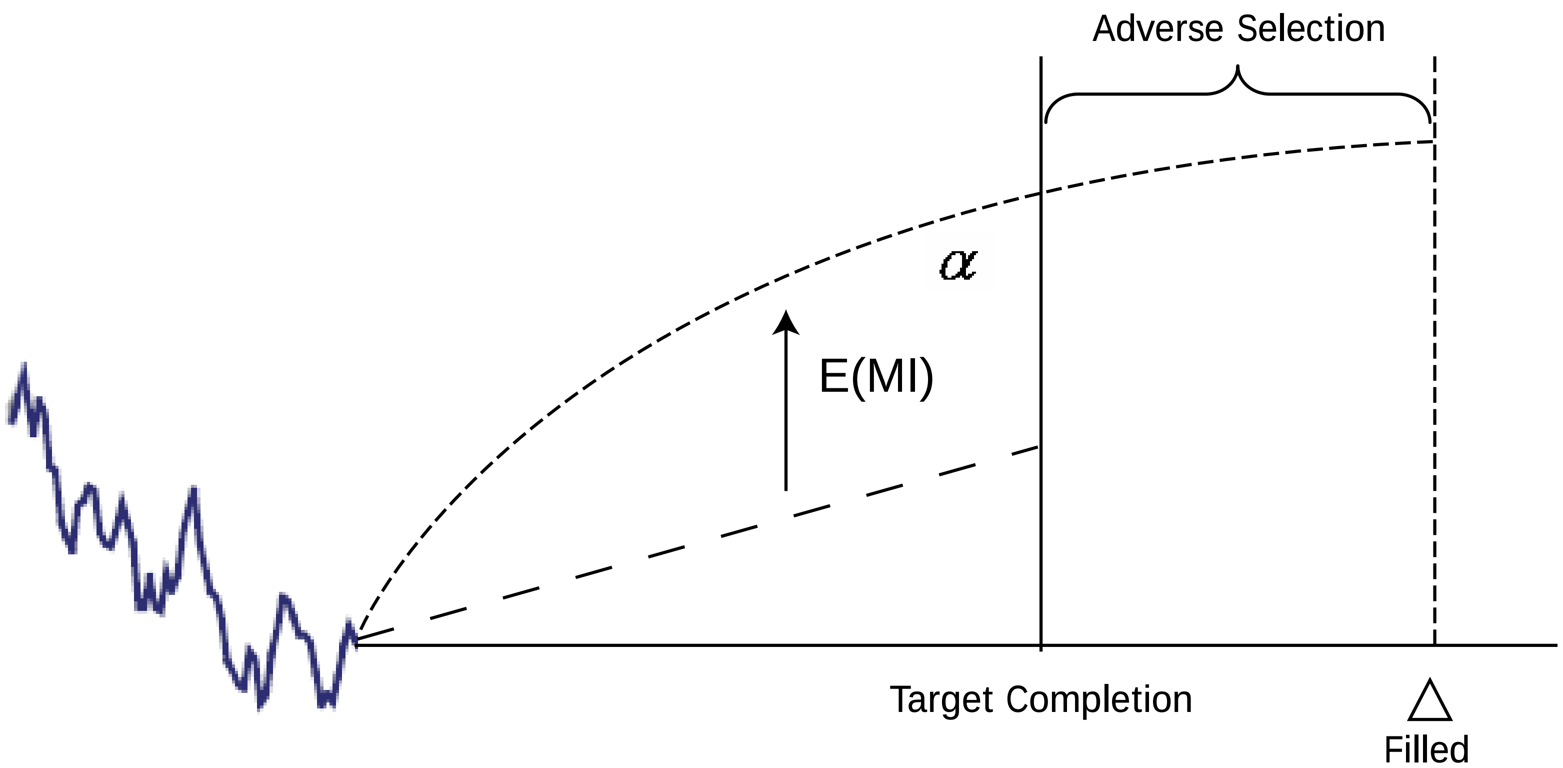
Order Execution Hierarchy



Implementation Shortfall

$$AS = \frac{1}{N}\sum_{k=1}^{N}\Theta\left(\ln\frac{P^k}{P^k_{Part}}\right)\ln\frac{P^k}{P^k_{Part}}, \qquad OS = \frac{1}{N}\sum_{k=1}^{N}\Theta\left(\ln\frac{P^k_{Part}}{P^k}\right)\ln\frac{P^k_{Part}}{P^k}$$

$$IS = E(MI \mid state) + AS - OS + \alpha$$



Market Impact

Martingale school:

- price is always the expected final price (no “good” or “bad” prices)
- therefore, the optimal schedule does not depend on price
- you cannot win or get “picked off”, i.e. *AS = OS*
- $\alpha = 0$

→ Only market impact matters: $IS = E(MI \mid state)$

KEY QUESTIONS

- Why is impact concave?
- Is permanent impact strategy-dependent?
- What is the optimal execution schedule?
- Are the incentives at a trading desk aligned with investment objectives?

Information Arbitrage Theory

Basic assumptions

1. Hidden orders can be detected in intervals of $\tau_i = \frac{1}{\pi_i{}^2}$ transactions of which $n_i = 1/\pi_i$ belong to the hidden order

2. Temporary impact depends on current velocity and total filled shares

$$\tilde{S}_{k+1} - S_k = -h_{k+1}(\pi_{k+1}, \xi_k) \qquad \xi_k = \sum_{i=1}^{k} n_i$$

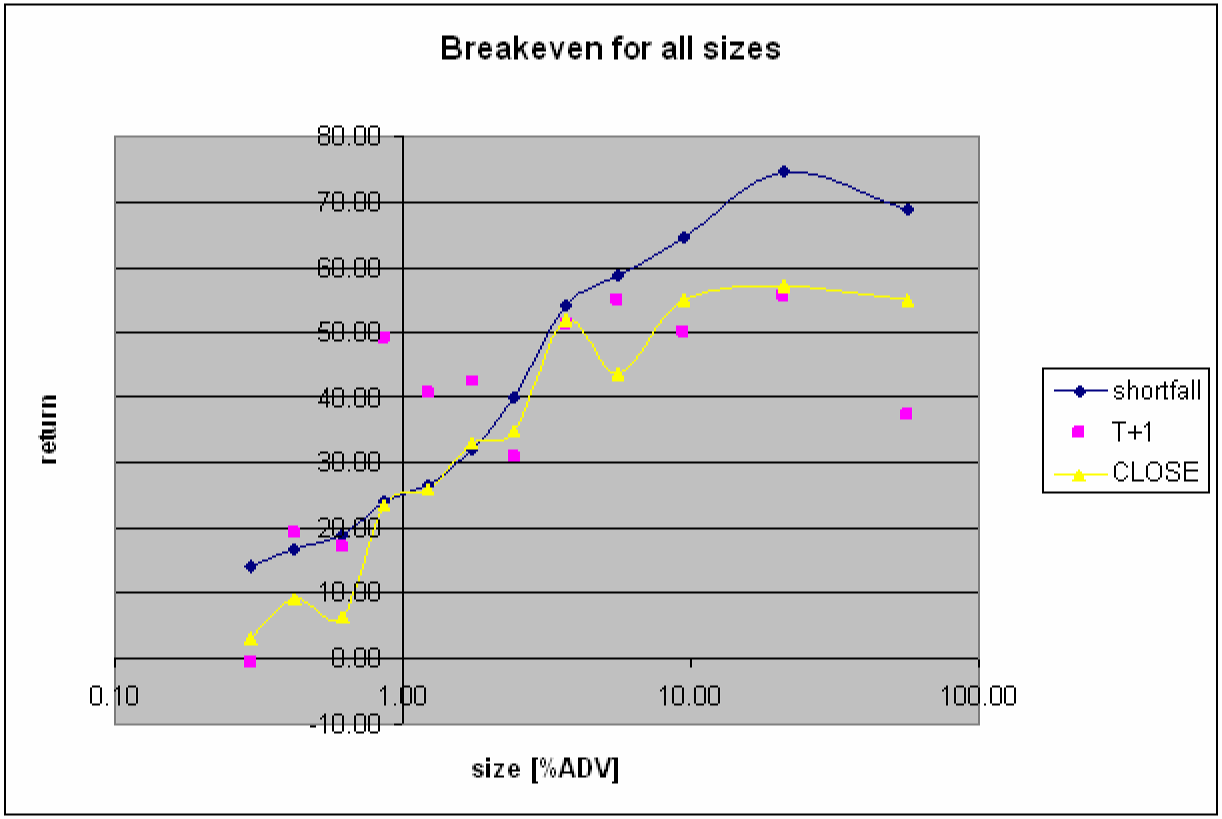
3. Efficiency: $p^+(i+1/_i)\,\langle r_i^+\rangle_G + p^-(i=N/_i)\,\langle r_i^-\rangle_G = 0,$

$$\langle r_i^+\rangle_G = \langle \tilde{S}_{i+1}\rangle_G - \langle \tilde{S}_i\rangle_G,$$

$$\langle r_i^-\rangle_G = \langle S_i\rangle_G - \langle \tilde{S}_i\rangle_G.$$

4. Hidden orders break even on average for every size: $S_k = \frac{\sum_{i=1}^{k} n_i \tilde{S}_i}{\sum_{i=1}^{k} n_i}$

Verifying the Breakeven Condition?



Solving the Theory

From $p^+({}^{i+1}/_{i})\,\langle r_i^+\rangle_G + p^-({}^{i=N}/_{i})\,\langle r_i^-\rangle_G = 0$, inserting breakeven we find,

$$\tilde{S}_{i+1} - \tilde{S}_i = \frac{p_i}{\Sigma_{i+1}}\left(\tilde{S}_i - \frac{1}{i}\sum_{k=1}^{i}\tilde{S}_k\right)$$

where $\Sigma_i \stackrel{\text{def}}{=} \sum_{k=0}^{\infty} p_{i+k} = p(N \geq i)$

Solving by recursion, $h_{i+1} = -\frac{1}{i\,p_1}\frac{\Sigma_1\,\Sigma_2}{\Sigma_{i+1}} r_1^+$

If hidden order sizes are Pareto-distributed and $r_1^+ \propto \pi_1^\beta$

$$p_i = \frac{1}{\zeta(\alpha+1,1)\,i^{\alpha+1}}$$

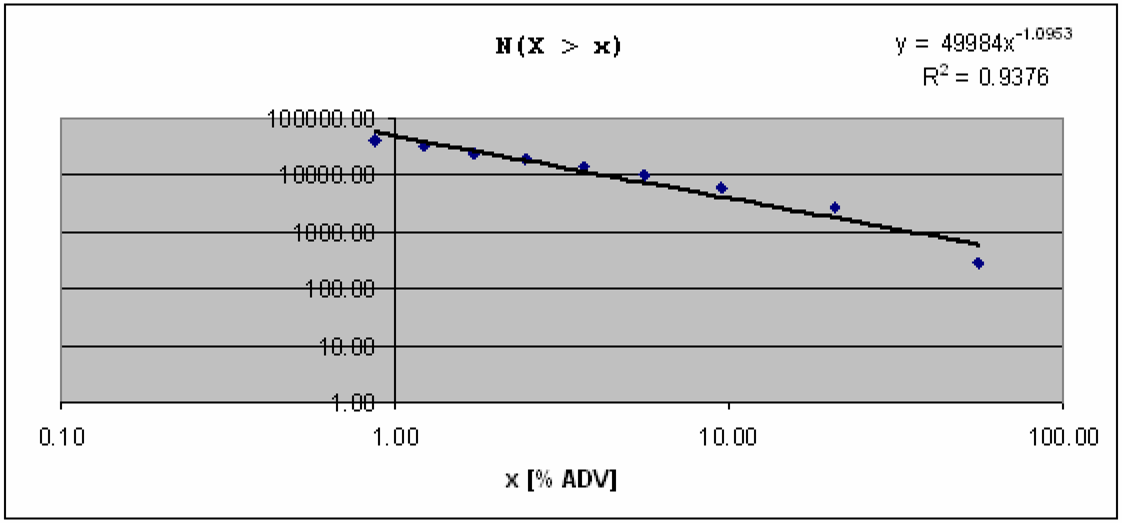
k >> 1

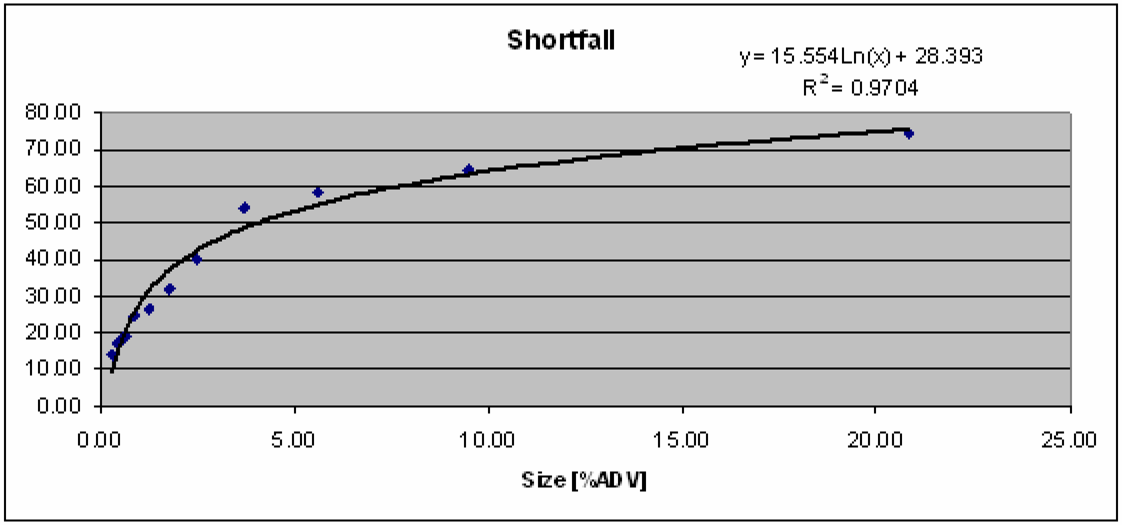
$$\begin{cases} \tilde{S}_k - S_{k-1} = \tilde{\mu}\pi_k^\beta\,|\,\xi_k\,|^{\alpha-1} & (\alpha > 1) \\ \tilde{S}_k - S_{k-1} = \tilde{\mu}\pi_k^\beta\,Ln(|\,\xi_k\,|) & (\alpha = 1) \end{cases}$$

Broker placements: $\alpha \approx 1.5$

Distribution of Institutional Order Sizes

Hidden order size distribution sets the average concavity of market impact





Optimization Objectives: 1. Shortfall

$$U(\vec{\pi}) \stackrel{\text{def}}{=} E(\vec{\pi}) + \lambda V(\vec{\pi})$$

$$X\, S_0 - \sum_{i=1}^{N} n_i\, \tilde{S}_i \stackrel{\text{def}}{=} E(\vec{\pi})$$

$$E(\vec{\pi}) = \sum_{i=1}^{N} X_i\, n\pi_i^{-1}\, \gamma_i + \sum_{i=1}^{N} n\pi_i{}^{-1}\, h_i$$

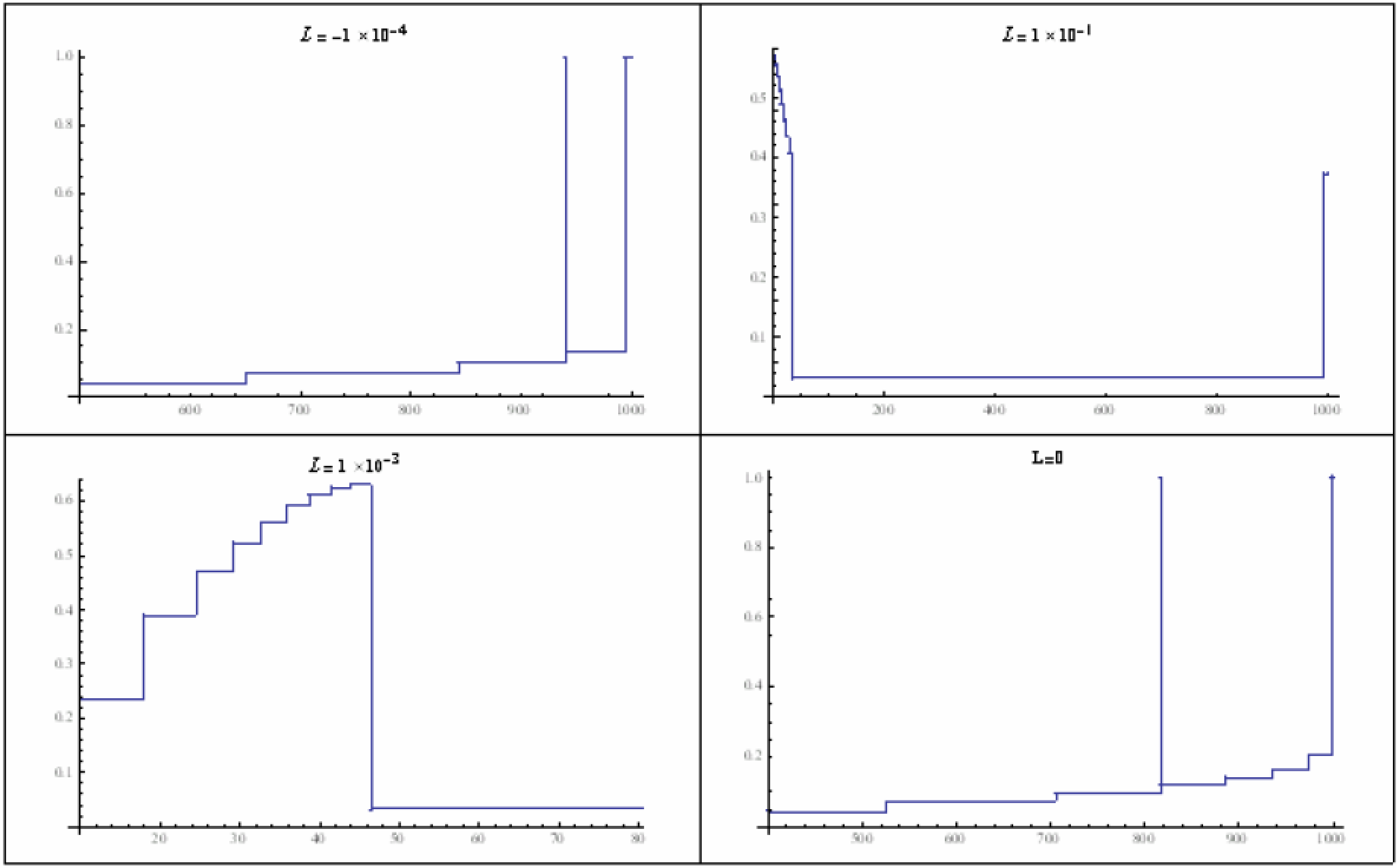
$$V(\vec{\pi}) = \sigma^2 \sum_{i=1}^{N} \frac{1}{\pi_i{}^2} X_i{}^2$$

Constraints

$$X = n \sum_{j=1}^{N} \pi_j^{-1}$$

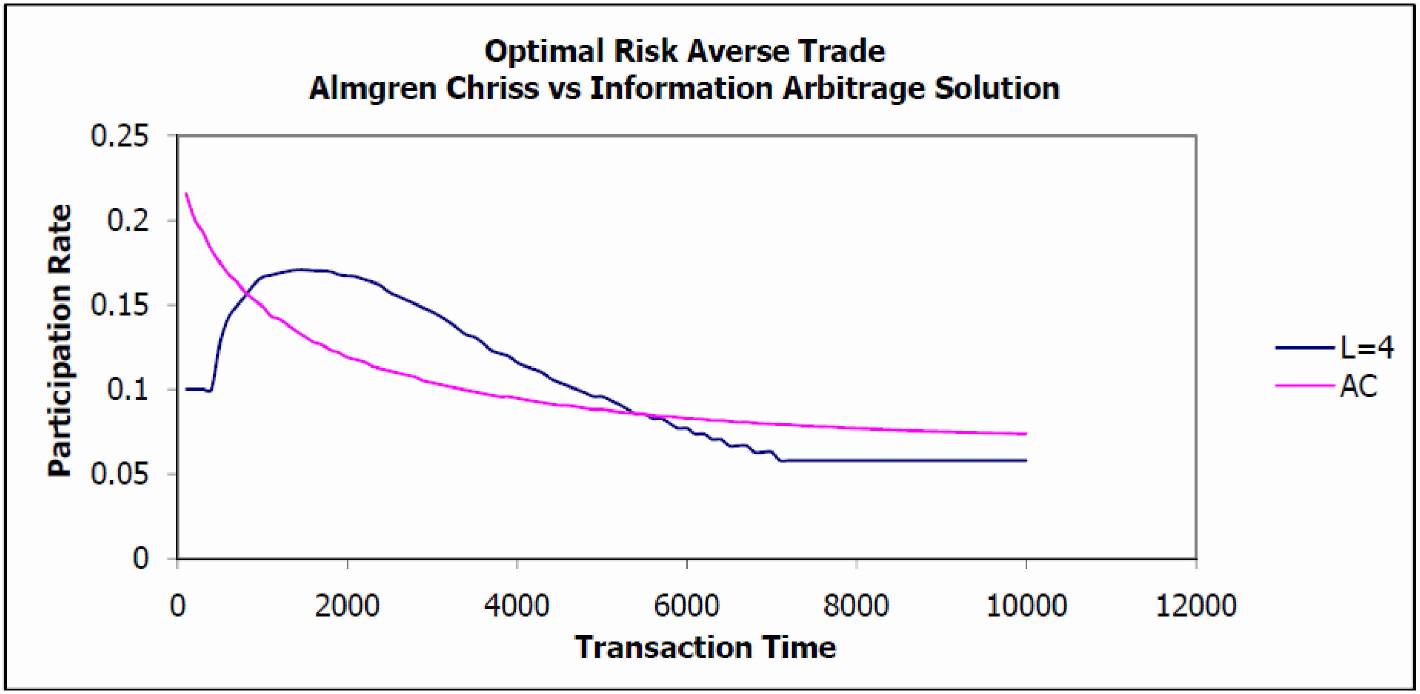
$$T \stackrel{\text{def}}{=} l_N - l_0 = \sum_{j=1}^{N} \pi_j{}^{-2}$$

Risk-adjusted Shortfall: Optimal Solutions



Risk-adjusted Shortfall: Comparing with AC 2000

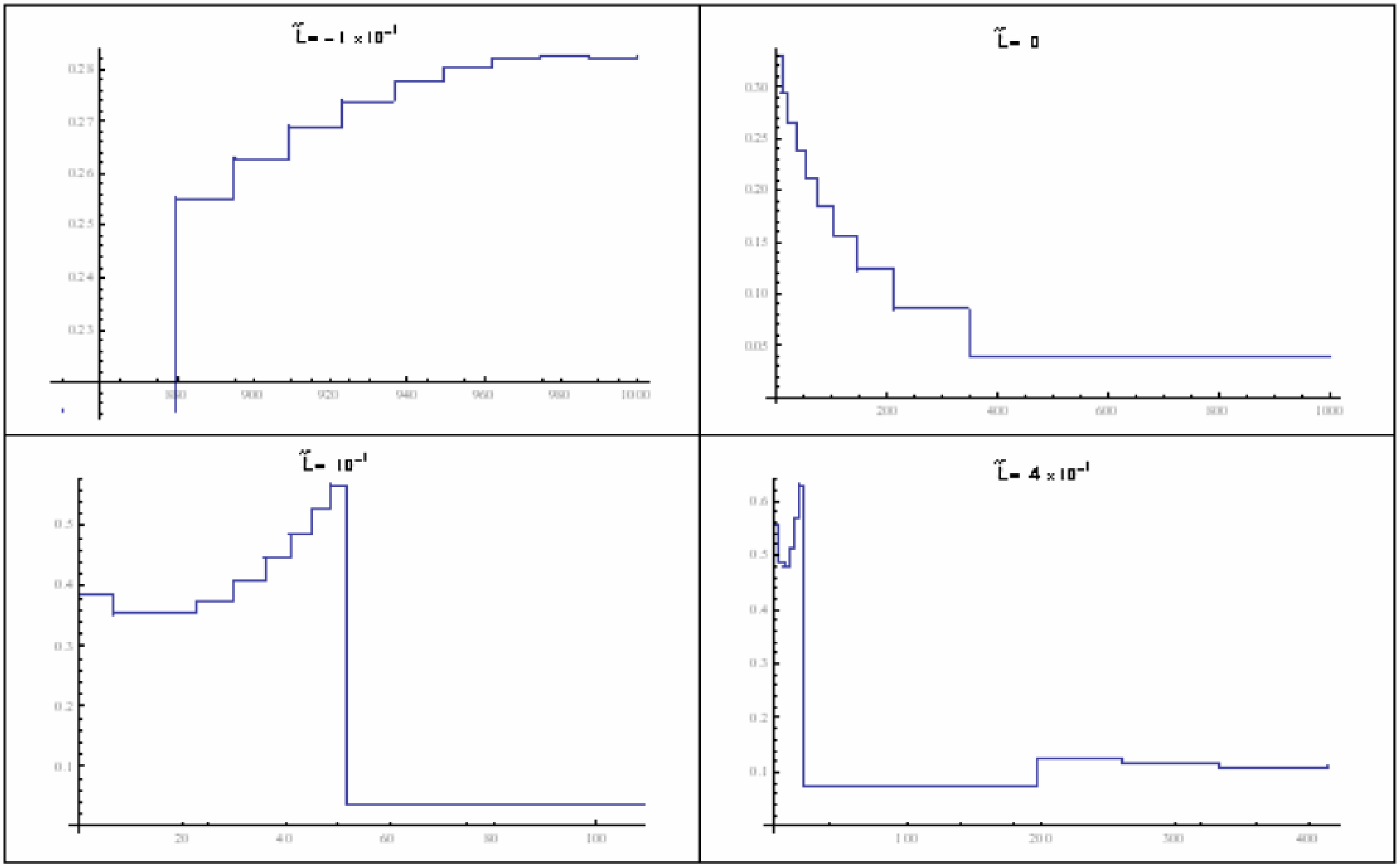
AC solution does not minimize risk-adjusted shortfall



Optimization Objectives: VWAP

$$\Delta(buy) := -\mathrm{VWAP} + \mathrm{capture} \overset{\mathrm{def}}{=} -\frac{1}{t_N}\sum_{i=1}^{N} \tau_i\, \widetilde{S}_i + \frac{1}{\xi_N}\sum_{i=1}^{N} \pi_i^{-1}\, \widetilde{S}_i$$

$$\langle\Delta\rangle + \tilde{\lambda}\, V = -\frac{\mu}{t_N}\sum_{k=1}^{N} \pi_k^{\beta-1}\, \xi_k^{\alpha-2}\, (\pi_k^{-1}\xi_k - t_k) + \tilde{\lambda}\, \sigma^2 \sum_{k=1}^{N} \pi_k^{-2} \left(\frac{\xi_k}{\xi_N} - \frac{t_k}{t_N}\right)^2$$



Optimization Objectives: a Frustrated Problem

Shortfall

L	Risk parameter, λ ($\$^{-1}$)	Traded shares	Shortfall per share ($\frac{\$}{share}$)	Shortfall, E ($)	Variance, $\sqrt{V}$ ($)
-1×10^{-4}	-3.49×10^{-5}	15650	0.16	2432.49	4786.66
0	0	19075	0.19	3535.74	6476.54
1×10^{-3}	3.49×10^{-4}	12500	0.20	2553.34	1337.96

VWAP

$\tilde{L}$	Risk parameter, $\tilde{\lambda}$ (share/$)	Traded shares	$\langle\Delta\rangle$ ($\frac{\$}{share}$)	Variance, $\sqrt{V}$ ($/share)
-1×10^{-1}	-8.73	15650	0.08	0.23
0	0	19075	-0.04	0.09
1×10^{-1}	8.73	12500	-0.03	0.03
4×10^{-1}	34.90	12500	-0.02	0.01

$\tilde{L}$	Traded shares	Shortfall per share ($\frac{\$}{share}$)	Shortfall, E ($)	Variance, $\sqrt{V}$ ($)
-1×10^{-1}	15650	0.17	2579.36	4725.47
0	19075	0.26	4967.87	3834.24
1×10^{-1}	12500	0.22	2729.20	1342.57
4×10^{-1}	12500	0.24	2935.89	2017.43

Conclusions

- Why is impact concave?
  - because only the unpredictable part of order flow causes market impact, and hidden orders are Pareto-distributed

- Is temporary impact strategy-dependent? -YES

- Is permanent impact strategy-dependent? – YES (it is the integral of t.i.)

- What is the optimal execution schedule with no alpha? – BACKLOADED

- Are the incentives at institutional desks aligned with performance – NO
  - most desks evaluate traders by a combination of VWAP and shortfall
  - pressure on explicit costs translates into more optionality in order flow and more adverse selection
  - both effects increase *implicit* costs

- Savings of 35% should be possible …

  … but it *is* a complex system and this is only a theory. We will see.