SDEs with price-feedback and power-law tails

Erice October 2009

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- Exchange insights and avoid wasted effort

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- Also, dependency is complex,
- Simple MV distributions will not do

Credit Crunch and Legacy

High profile people were moaning about the difficulty of modelling repeated 25-sigma events in major markets. It's only unlikely if you try to stay in a Gaussian world.

We also had poor dependency modelling in many areas of finance.

Investors appreciate need to allow for non-Gaussian returns and non-Gaussian dependency. They might have all kinds of risk and return goals, based on MV, Utility, Omega etc.

There is also blame management. Big topic but not for here.

Talk Focus

This talk will be largely speculative and focus on the following issue. The triggering of trades in the financial markets is often **highly price-sensitive**. Traditional technical-trading, price-sensitive best execution and computer-based algorithmic trading all induce **price-feedback**. The details can be found at:

`http://arxiv.org/abs/0811.0182`

and related statistical dependency aspects are discussed in "Dependency without copulas or ellipticity", with A. Munir, *Eur J Finance*.

`DOI:10.1080/13518470802697402`

Refs ctd

The properties of “Pearson Diffusions” have been studied recently by Forman and Sørensen 2008. The Pearson diffusions: A class of statistically tractable diffusion processes. *Scandinavian Journal of Statistics*, Vol. 35, Number 3, September 2008 , pp. 438-465(28).

We are still tracking back: the idea of a Pearson diffusion for fat tailed returns was certainly proposed by Yuichi Nagahara by 1996. Non-Gaussian Distribution for Stock Returns and Related Stochastic Differential Equation, *Financial Engineering and the Japanese Markets*, 3, 121-149. An SDE and integral formula for a density dates to *E. Wong in 1953!* Very hard to use Wong pdf. More insight here.

Perspectives on Chartism

Professor Pietronero made many very interesting remarks on the inclusion of technical trading, or chartist trading. He should say such things things more often to financial mathematics community. They write off the chartists and miss an important point - technical trading exists and is BIG.

While I do not agree that the idea of switching between fundamental and chartist is the right mechanism in detail, I applaud the inclusion of the ideas in general. I think the switching that matters is between technical traders in “momentum” mode and ”mean-reverting mode”. We will try to capture this in a model whose underlying expression is very simple.

My own route to these ideas

Began with some work to understand the routine construction of quantile functions (functional inverses of cumulative distribution functions) for the direct simulation of exotic distributions. After some direct algebraic work with the Student case, I began a collaboration with György Steinbrecher on the direct representation of quantile functions as solutions of non-linear ordinary and partial differential equations. We called this idea “Quantile Mechanics” and it put the construction of many key quantiles on the same footing as the construction of traditional special functions, cf Legendre’s equation as arising from Laplace and Shroedinger equations, solutions by series.

Quantile Mechanics summary

2008. Quantile Mechanics, *European Journal of Applied Mathematics*, **19** (2), 87-112.

$$F'(x) = f(x),\ F(Q(u)) = u$$

$$Q'(u) = 1/f(Q(u)),\ Q'' = H(Q)(Q')^2$$

$$H(Q) = -\frac{\partial}{\partial x}\log(f(x))|_{x\to Q}$$

e.g. normal case

$$Q'' = Q(Q')^2$$

$$Q(1/2) = 0,\ Q'(1/2) = \sqrt{2\pi}$$

And similarly for Student, others....

Quantile Mechanics II

Time dependent problems arising from SDEs.

$$dx_t = \mu(x_t, t)dt + \Sigma(x_t, t)dW_t \ ,$$

Form quantile form of Fokker-Planck equation (J.A Carrillo, and G. Toscani 2004)

$$\frac{\partial Q}{\partial t} = \mu(Q, t) - \frac{1}{2}\frac{\partial \Sigma^2}{\partial Q} + \frac{\Sigma^2(Q, t)}{2}\left(\frac{\partial Q}{\partial u}\right)^{-2}\frac{\partial^2 Q}{\partial u^2} \ .$$

So you can make an SDE with a target equilibrium density by equating:

$$\frac{Q''}{Q'^2} = H(Q) = -\frac{\partial}{\partial Q}\log(f(Q)) = \frac{2}{\Sigma^2}\left(\mu - \frac{1}{2}\frac{\partial \Sigma^2}{\partial Q}\right)$$

Context

The development of that is another story. I bring it up here to explain how I got to this point, but also to point out that this is also in a sense in the spirit of econophysics. We are solving a problem in probability targeted at Monte Carlo simulation by deploying technology use in old-fashioned theoretical physics - solving differential equations. We even do it by series, though obtain some interesting non-linear recursions (quadratic for normal, cubic for Pearson family). We can solve these symbolically or numerically, then e.g. create fast rational approximations for live use. With DEs we can do changes of variables, and explain things like Cornish-Fisher expansions, and make better algorithms for GPU (see e.g. ArXiv 0901.0638).

However

It should nevertheless be appreciated that many non-Gaussian distributions have an easy origin as *equilibria* of simple SDES. Known and rediscovered many times. We can get Normal, Student, Gamma, Beta, Pearson Type IV within standard SDE theory. Am open-minded about exotic modifications, e.g. in style of Tsallis-Borland, forcing in time-dependent Student T. We seek simple theory out of which such distributions may emerge, without this, or by changing form of CLT. While extremely interesting, there is another simpler route, which also will suggest an intriguing alternative extension of Student to negative degrees of freedom. There is no unique analytic continuation and the proposed dynamics comes up with something else....

Acknowledgments

- G. Steinbrecher - hybrid processes in physics
- K. Vanguelov KCL, M. Schofield, A. Macrina KCL
- R. Wilson and A. Healey, Nomura
- M. Sørensen, Y. Nagahara for useful correspondence
- M. Yor for elucidation of some of the mathematics

Fat Tails Levels 1 and 2

Level One is just getting them into routine risk management, without necessarily tying up all the issues it generates with option pricing.

Level Two is about having a comprehensive tie up, within each asset class, of observed data, basic maths, option pricing and risk management.

Level Two is classical applied maths view - you are modelling reality in self-consistent manner, though it is the human financial world, not the physical one - predictions are distributional.

There is no excuse for the past R&B failure to implement level one.

Level 1: What do we have to do!!!

Fergusson & 2006 (AMF). Daily index log-returns MLE Student T_4 amongst hyperbolic distributions. Relation to BU/MIT study? FP work considered Generalized Hyperbolic Distribution and did MLE amongst that. Found T_4. CDF tail x^{-4}. See paper in EPJ-B also.

Student is one of many conditionally Gaussian distributions. W. Gosset derived it in 1908 essentially using a variance that is inverse gamma, in his case arising as a sample-estimated variance. This is also the real key to its multivariate extension(s). There are many others. Equilibrium of Heston CIR SDE for variance is gamma, so asset returns are conditionally log-normal with gamma variance.

What do we have to do 2

For Student T: I publicised the older methods in survey Shaw, J Comp Fin 2006. All known bar ratio of uniforms.

Ralph Bailey (1994): how to modify Box-Muller/polar form to move from Gaussian to Student T. ONE LINE OF CODE to change in a widely used banking algorithm. Also $\nu < 0$.

Hill (1970) produced approximations for Student quantile. JCF 2006 produced some closed form quantiles (1,2,4) and series (3...) Quantile Mechanics gives us simulation for all $\nu \geq 1$.

Hyperbolic, Stable forms being pursued by many colleaues. But Student pragmatic and in data. Again, need Stanley-Platen reconciliation.

Yet, in June 2009

In the FT of 10th June 2009, Lord Turner, Chair of the FSA, is quoted as follows:

The problem, he said, was that banks' mathematical models assumed a "normal" or "Gaussian" distribution of events, represented by the bell curve, which dangerously underestimated the risk of something going seriously wrong.

While there is always the unpredictable tsunami event outside scope of historical data (the excuse), even the routine modelling from history was wrong. A 25 sigma event is over 10^{130} times more likely in T_4 than in Gaussian. Lord T did say "bank's" - not FM. So perhaps we are getting somewhere.

Level 2: Short term returns

The *reality* is that there are many types of trader. I will focus on two: fundamental and technical.

F-traders might have a buy or sell order based on a portfolio rebalance, tilt towards high yield, low P/E etc. Apart from demanding efficient execution on the day, the desire to trade is set in advance. Then there are the technical traders, who react to price movements in many different ways, considered by some to be irrational. T-trades are present where we understand them or not! They depend on price.

Fund and Tech Model

I have built a model of the combined process, starting with a Poisson model of trader arrival and allowing for price-sensitive and price-insensitive trades, and a crude price impact model.

Details are in the ArXiv preprint.

Here I will skip to the key mathematical reductions. We can have various mixtures of trade sizes. Some large isolated trades can indeed generate jumps, but even without that we get some interesting systems as follows.

The governing SDEs

Making assumptions (including linearizing a lot), we are lead (in the absence of jumps) to the following for intraday. Let $X_t = \log(S_t/S_0)$. Then, for some f, g with $f(0) = g(0) = 0$,

$$dX_t = (\mu_1 - f(X_t))dt + \sigma_1 dW_{1t} + g(X_t)dW_{2t}$$

With one set of simplifying assumptions, we are led to (technical trade arrival rate fixed, further linearization, Schofield)

$$dX_t = (\mu_1 - \mu_2 X_t)dt + \sigma_1 dW_{1t} + \sigma_2 \sqrt{|X_t|}dW_{2t}$$

This is a hybrid ABM-CIR process. Investigation under way.

The ABM-GBM Hybrid

The form I am going to describe today has further volatility enhancement in the technical component:

$$dX_t = (\mu_1 - \mu_2 X_t)dt + \sigma_1 dW_{1t} + \sigma_2 X_t dW_{2t}$$

where the arithmetical terms arise from the fundamental (price insensitive) trades and the geometric terms arise from technical (price-sensitive) trades. W_{it} are standard BMs.

I first saw this in recent plasma physics literature (GS), but such hybrids go back some way in statistical physics lit. (see refs). Many here know more of this than I, Langevin eqns.

The link to Pearson-Diffusions

If ρ is the correlation between the two Brownian motions, then we can write the SDE as

$$\begin{aligned} dX_t =& (\mu_1 - \mu_2 X_t) dt \\ & + \sqrt{\sigma_1^2 + X_t^2 \sigma_2^2 + 2\rho\sigma_1 X_t \sigma_2^2}\, dW_t \,. \end{aligned} \tag{1}$$

This is one of the class of “Pearson diffusions” considered by Nagahara, and Forman and Sørensen. This sub-family generates Pearson Type IV and Student.
This is not new with me, but I hope to persuade you of some insights.

Contribution

We have made progress in

- Equilibrium analysis
- SDE essentials
- Moment analysis
- Dynamic solution of Fokker-Planck
- Classification of market states
- ID of both equilibria and explosive panic

Need to complete FP analysis, Var, CVar etc and calibration tools

Equilibria?

Simplify and set $\mu_1 = 0 = \rho$, so that we obtain the SDE

$$dX_t = -\mu_2 X_t dt + \sqrt{\sigma_1^2 + X_t^2 \sigma_2^2}\, dW_t \ . \qquad (2)$$

If an equilibrium exists (necessary that $\mu_2 > 0$), we obtain a Student t distribution with degrees of freedom

$$\nu = 1 + 2\frac{\mu_2}{\sigma_2^2} \ . \qquad (3)$$

We have made our fat-tailed (excess kurtosis) model from a natural price-sensitive model. *There is no need for $\nu > 0$* - depends on MR-MOM balance.

SDE Essentials

What have we really got here? Let

$$X_t = \frac{\sigma_1}{\sigma_2} \sinh(Z_t) \ . \tag{4}$$

$$dZ_t = -\frac{\sigma_2^2}{2} \nu \tanh(Z_t) dt + \sigma_2 dW_t \ . \tag{5}$$

Now rescale time, we get the reduced form:

$$dZ_\tau = -\frac{1}{2} \nu \tanh(Z_\tau) d\tau + dW_\tau \ . \tag{6}$$

It is “hyperbolic OU” (Wong). The mean-reversion weakens giving us fat tails. There is one essential parameter: ν, real, either sign.

Variance Explosion

Special case $\mu_1 = \rho = 0$, variance $V(X_t)$:

$$V(X_t) = \frac{\sigma_1^2}{\sigma_2^2(\nu - 2)} \left[1 - e^{-\sigma_2^2(\nu-2)t} \right] \sim \sigma_1^2 t + O(t^2) \; , \tag{7}$$

Market starts off Gaussian. Then dynamics critically dependent on sign of $\nu - 2$. If strength of mean-reverting trades is such that $\mu_2 > \sigma_2^2/2$ the market settles down. If instead $\mu_2 < \sigma_2^2/2$ the variance grows exponentially. There is a region $0 < \mu_2 < \sigma_2^2/2$ where the average level stabilizes but the variance does not. If $\mu_2 < 0$ both the average level and variance grow exponentially. If $\nu > 2$ the distribution settles down to the Student equilibrium.

When 25σ is common

If $\nu < 2$ then with $\alpha = \sigma_2^2(2-\nu) > 0$, then

$$\frac{V(t)}{\sigma_1^2 t} = \frac{1}{\alpha t}\left[e^{(\alpha t)} - 1\right] \qquad (8)$$

Then if $\alpha > 6.48$ this is > 100, so the variance is 100 times bigger than you thought for this time based on the Gaussian model, and Gaussian 25σ event is really only a 2.5 Sigma event if you use the right Sigma.

Dynamic Student Distributions

The solution of the Fokker-Planck equation is hard. But it pays off even from some special cases. I have Laplace transform solution for case $\rho = \mu_1 = 0$, and some inversions that I will now share.

We can now see just how deceitful a Gaussian view of markets can be.

The PDEs and transform ODEs are all in the paper with details - I will discuss results for $\nu = 0$ and $\nu = \pm 2$. There are some pretty links with physics as it is all a disguised form of the associated Legendre equation: $L = \nu/2$ is the quantity in physics normally associated with angular momentum; matters are easier (not quantized!) with $L \in \mathsf{Z}$.

Dynamic T: $\nu = 0$

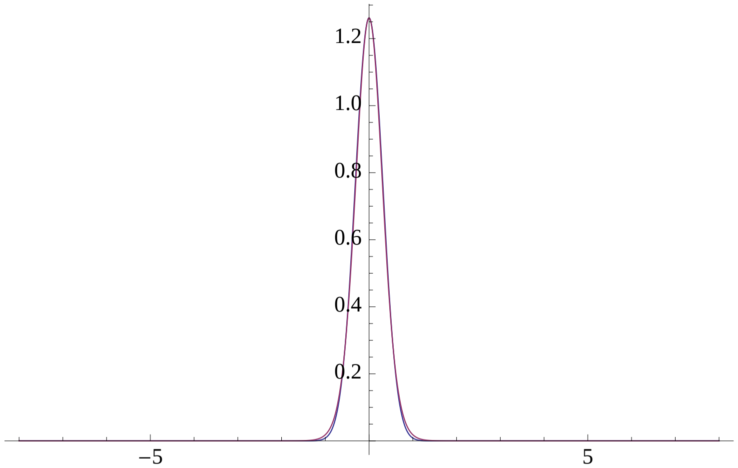
The PDF is (this is the one easy case of "hyperbolic OU" - Gaussian with a hyperbolic transformation)

$$f(x,t) = \frac{1}{\sqrt{2\pi t(\sigma_1^2 + \sigma_2^2 x^2)}} \times \exp\left\{ \frac{-1}{2\sigma_2^2 t} [\sinh^{-1}(\sigma_2 x/\sigma_1)]^2 \, , \right\} \tag{9}$$

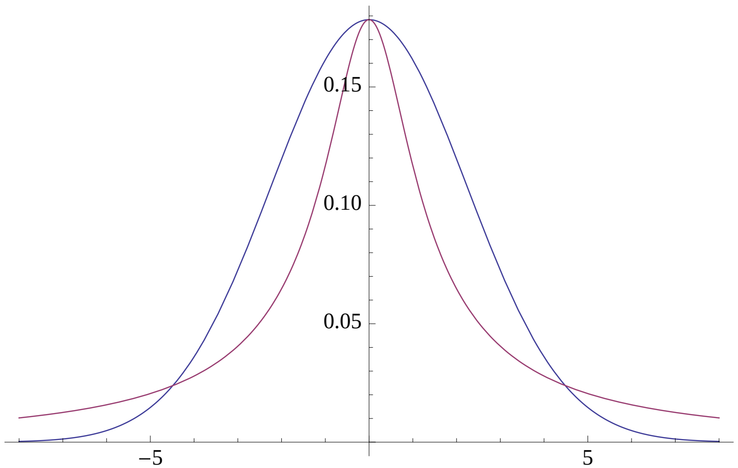
Compare with Guassian with variance $\sigma_1^2 t$.

Density vs Gaussian

Early times:



Later: momentum trades fatten density:



The deceit...

At the start of the trading period the distribution of log-returns is barely distinguishable from Gaussian. However, as time passes the hybrid distribution spreads out more, in a manner consistent with the variance explosion formula. At *all* times the probability of a very small move remains at the Gaussian level - the PDF osculates the Gaussian at the origin. The overall behaviour represents the hidden menace of these processes. It starts off looking Gaussian with variance $\sigma_1^2 t$; the probability of a very small movement remains near the Gaussian value, yet dependent on the size of σ_2 the probability of extreme movements grows exponentially in time.

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- It tends to an equilibrium Student T_2 distribution with infinite variance and kurtosis
- Late time behaviour has $O(x^{-3})$ PDF tails.

$\nu = 2$ Chameleon PDF

This odd PDF is written down in the paper - it's just the solution for the case $\nu = 2$. It starts off looking like

$$f(x,t) \sim \frac{\exp[-\frac{x^2}{2t\sigma_1^2}]}{\sqrt{2\pi t}\sigma_1}\ , \tag{10}$$

and tends to

$$f(x,t) \sim \frac{\sigma_2 \sigma_1^2}{2\left(\sigma_1^2 + x^2\sigma_2^2\right)^{3/2}}\ , \tag{11}$$

General PDF messy, but arises from the simple SDE

$$dX_t = -\frac{\sigma_2^2}{2} X_t dt + \sqrt{\sigma_1^2 + \sigma_2^2 X_t^2} dW_t \tag{12}$$

States of Market

We started of with four parameters:

- σ_1, the fundamental volatility;
- σ_2, the technical volatility;
- μ_1, the fundamental drift;
- μ_2, the technical drift;

μ_1 has not been treated but its interpretation is clear. A critical parameter is:

$$\nu = 1 + \frac{2\mu_2}{\sigma_2^2}\,. \tag{13}$$

States of Market II

If an equilibrium is achieved, ν is the degrees of freedom of the associated Student distribution that results. But now we see that it plays an esential dynamical role:

- if $\nu < 2$ the variance explodes exponentially;
- if $\nu = 2$ the variance remains in Gaussian form, but any member of the chameleon family may exist;
- $\nu > 2$ the variance tends to a constant.

States of Market III

Volatilities play a different role.

- $\nu = 1 + \frac{2\mu_2}{\sigma_2^2}$ - partially determines the market condition;
- σ_2 defines the time-scale;
- σ_2/σ_1 defines the asset price scale.

When $\mu_1 = 0$, we can make changes of variable to get

$$dZ_\tau = -\frac{\nu}{2}\tanh(Z_\tau)d\tau + dW_\tau \ . \tag{14}$$

with all other parameters removed by transformation. We have explicit time-domain solutions for the resulting PDF for $\nu = 0, \pm 2, 4$ and the Laplace transform for other values.

Momentum-dominated markets

In August I gave an earlier form of this talk in Kyoto and Marc Yor pointed out that the ABM-GBM hybrid could be formally solved in terms of the integral of the exponential of one Brownian motion against another. The simple hyperbolic sine solution we found is another expression of the Bougerol identity, that

$$\int dW_{1t} \exp(W_{2t})$$

is distributed as $\sinh(W_t)$. This is what I think is the Student with zero degrees of freedom - it is essentially time dependent! There is not enough mean reversion. What if there is a serious level of momentum trading....

ADY Generalized Bougerol

Earlier, Alili, Dufresne and Yor had found one generalization which is in fact a $\nu = -2$ solution in our notation. This distribution can be written down in terms of densities that look like a dispersive solution of the wave equation, with a pair of waves travelling in opposite directions. The ADY variable gives us

$$\sinh(W_t + Xt)$$

where X is Bernoulli!

Such a momentum-dominated market picks up a direction at random and sails off in that direction. We think this behaviour is generic, at least asymptotic for $\nu < 0$ and are working on proving it.

Conditional Gaussian aspects

The PDF for objects like

$$\int dW_{1t} \exp(W_{2t} - \nu t/2)$$

is conditionally Gaussian with variance given by integrals of exponentials of ABM - these are the variables of Asian options. Another Gaussian mixture. The conditioning distribution of variance corresponds is that of the random variable

$$V = \sigma_1^2 \int_0^t \exp[2\sigma_2 \tilde{W}_{2u} - \nu \sigma_2^2 u]$$

At late times this tends to inverse gamma (Dufresne).

Student T with -ve dof

When there is mean reversion power law decay emerges easily via a Student equilibria. The chameleon point $\nu = 2$ is a transitional point. Below that all hell breaks out as then there is a further fattening and then a momentum-dominated panic for $\nu < 0$. We think this is a natural extension to negative degrees of freedom.
Mathematica demo for $\nu = -4$...

Summary

- Model mix of fundamental & technical trades;
- Hybrid A-G-BM in absence of jumps;
- Rich variety of market states;
- Days which start off Gaussian will not usually remain that way;
- Variance explosion a key feature;
- Fat tails emerge in response to simple composite model of trader behaviour;
- Negative ν gives momentum panic.

We need risk and calibration tools as well. VaR, tail VaR look tractable. Computational Issues, ABM-CIR form under investigation.

Summary

The financial mathematics community has historically dismissed technical trading as witchcraft. But it is carried out, whether we believe it or not. Allowing for it in a model that is simple in its basic form leads to dramatically different qualitative and quantitative behaviour. With mean-reversion we get fat-tailed equilibria, but also an interesting momentum dominated set of states.

This has just used a basic price-feedback mechanism that could have been taking from many areas of physics, engineering or biology...

We have work to do. But we have a pretty Laplace FP equation that is Legendre equation with $\nu = 2L$, and some new solutions to illustrate behaviour.