Financial Bubbles Analysis with a Cross-Sectional Estimator

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October 26, 2009

Outline

Setup

- Given N stocks on a market place, a reference date t_{ref} and the current date t

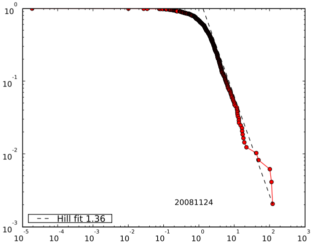
$$S_N(z) = \frac{1}{N}\sum_{i=1}^{N} \mathbb{1}_{\{X_i(t_{ref},t)>z\}}$$

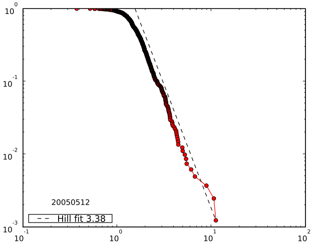
with $X_i(t_{ref}, t) := \frac{S_i(t)}{S_i(t_{ref})}$

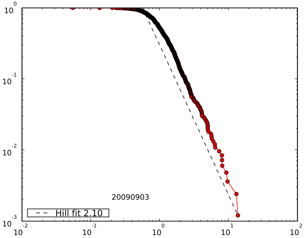
- Two statistical regularities:
  1. $S_N(z) \sim z^{-\alpha}$, for $z \sim +\infty$, with $\alpha \searrow 1$ during the bubble
  2. $V_N = -\int_0^{+\infty} z^2 \mathrm{d}S_N(z) - \left(\int_0^{+\infty} z\,\mathrm{d}S_N(z)\right)^2 = \frac{1}{N}\sum_{i=1}^{N}\left(X_i - \bar{X}_N\right)^2$ increases dramatically before crashes
- V_N is a measure of dispersion: $V_N = \frac{D_N}{2} = \frac{1}{2N^2}\sum_{i,j=1}^{N}\left(X_i - X_j\right)^2$.

Real data illustration: power-law behavior

AORD:500 stocks, NYA:1800 stocks, SSE:900 stocks, $t_{ref} = 01/01/2003$

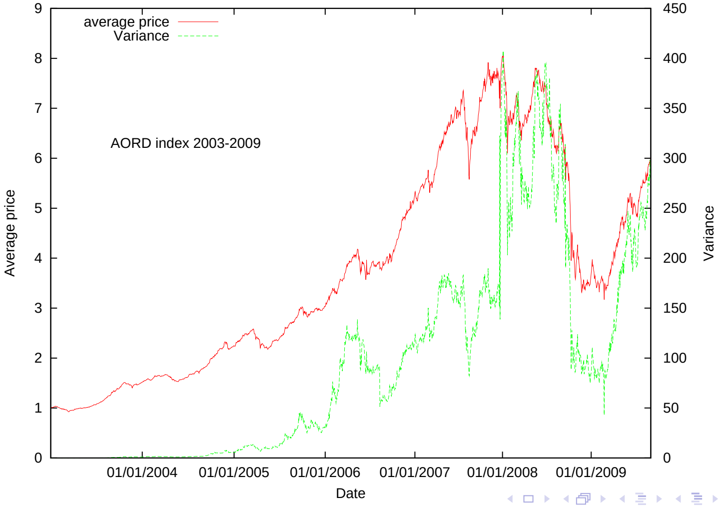






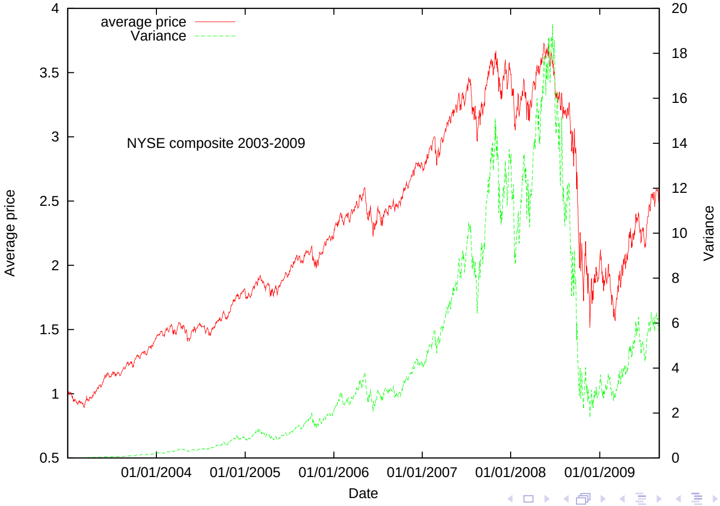
Real data illustration: evolution of V_N

AORD, 500 stocks



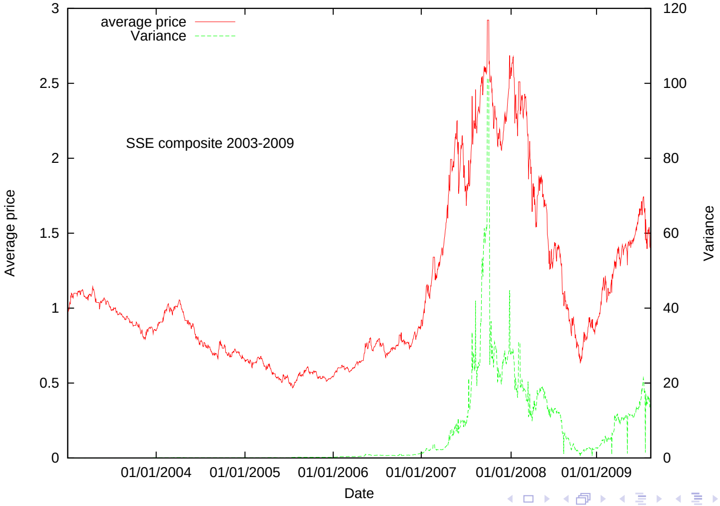
Real data illustration: evolution of V_N

NYA, 1800 stocks



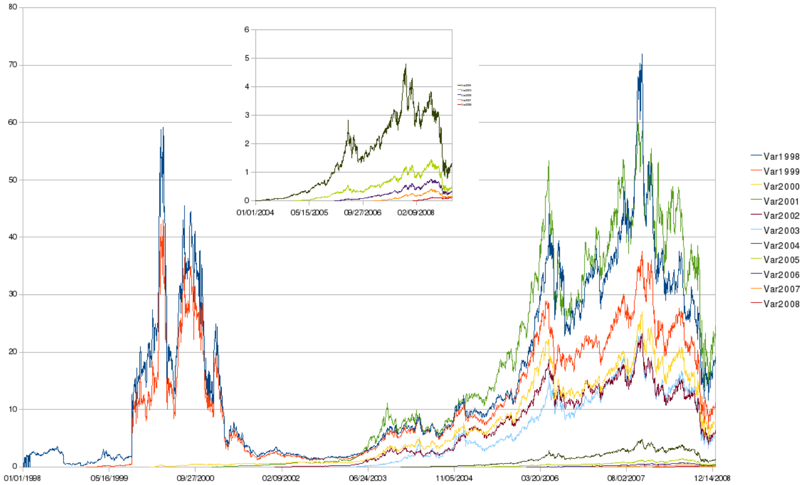
Real data illustration: evolution of V_N

SSE, 900 stocks



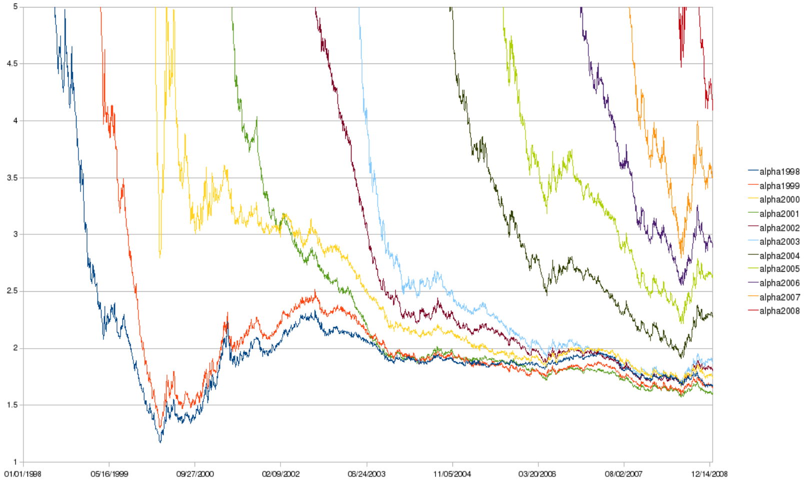
Robustness w.r.t. t_{ref}

Russell 3000, 3000 stocks



Robustness w.r.t. t_{ref}

Russell 3000, 3000 stocks

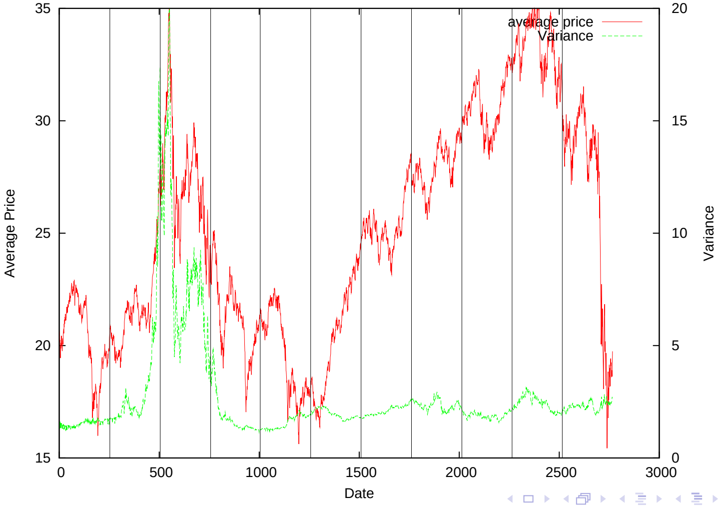


Size effect

- $\forall i \; S_i(t) = \lambda S_i(t-1) \Longrightarrow V_N(t) = \lambda^2 V_N(t-1)$
- Potential normalizations:
  - $\tilde{X}_i(t_{ref}, t) = \frac{X_i(t_{ref}, t)}{<X(t_{ref}, t)>} \Longleftrightarrow$ use $\tilde{V}_N(t_{ref}, t) = \frac{V_N(t_{ref}, t)}{<X(t_{ref}, t)>^2}$
  - $\tilde{X}_i(t) = \frac{S_i(t)}{<S(t)>} \Longrightarrow$ we get rid of t_{ref}

Size effect

Russell 3000, 3000 stocks



Link with the variance-covariance structure

- Analytical link with the variance-covariance structure

$$\mathbb{P} - \lim_{N\to+\infty} V_N = \lim_{N\to+\infty} \left(\frac{1}{N}\sum_{i=1}^{N}\sigma_i^2 - \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{ij}\sigma_i\sigma_j + \frac{1}{N}\sum_{i=1}^{N} m_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} m_i\right)^2 \right)$$

where $m_i =< X_i >$, $\sigma_i^2 =< X_i^2 > - m_i^2$ and $\rho_{ij} = \frac{<X_iX_j> - m_im_j}{\sigma_i\sigma_j}$

- Two phases: from the bubble to the crash
  1. during the bubble: for some (i,j) $\rho_{ij} \searrow \implies V_N \nearrow$
  2. during the crash: $\rho_{ij} \nearrow \implies V_N \searrow$

Recap and further research

Recap

- a bubbling market dispersion indicates bubbles
- market dispersion estimation is quite robust w.r.t. the reference date
- bubbles can be related to the variance-covariance structure of financial stocks

Further research

- estimate m_i, σ_i, ρ_{ij} from HF data
- investigate intraday bubbles
- suggest an exchange model

Thank you for your attention!

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