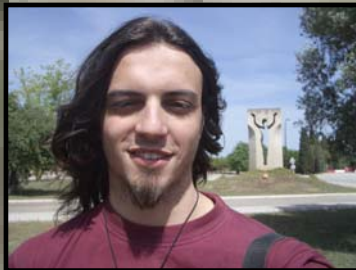


# *Language Competition: an example of Social Consensus*



**Xavi  
Castelló**



**Victor M.  
Eguíluz**



**Federico  
Vázquez**



**Riita  
Toivonen**



**Maxi  
San Miguel**



+ Dietrich Stauffer, Lucía Loureiro



{ J. Saramaki  
K. Kaski

***This presentation featured a number of movies and numerical demonstrations: This and related material is or will be posted in the web page:***

***<http://ifisc.uib.es/eng/lines/social.html>***

# ***What opinions, cultural traits and language have in common?***

***Language competition:** dynamics of language use and learning due to social interactions, modeled in a network of social interactions*

*subject of study: society*

## MODELS of SOCIAL CONSENSUS:

Determine when the dynamics of a set of interacting agents that can choose among several **options** (*political vote, opinion, cultural features,...*) leads to a **consensus** in one of these opinions, or when a state with several **coexisting** options prevails.

## CONSENSUS PROBLEMS:

Dynamics of language competition

### INTERACTIONS: Mechanisms and Network

-VOTER MODEL

#### MODELS

-SPIN FLIP KINETIC ISING MODEL (T=0)

-AXELROD MODEL

-GRANOVETTER'S MODEL

-Imitation

#### MECHANISMS

-Following majority. Social pressure

-Homophily

-Threshold for social pressure



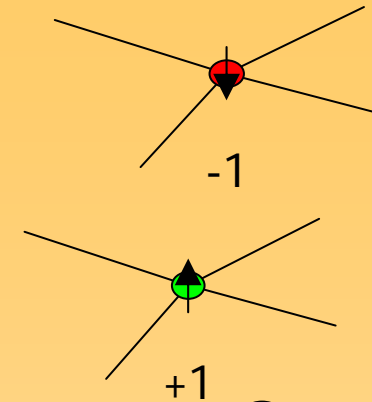
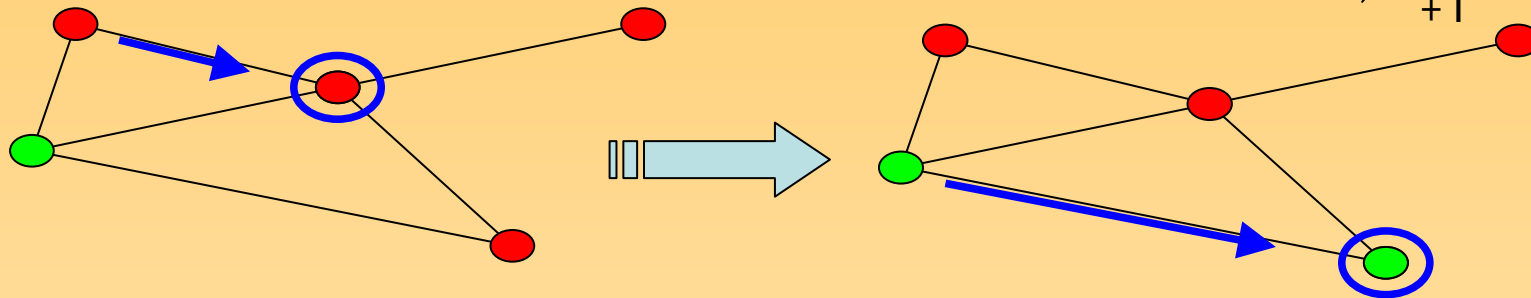
**IFISC**



<http://ifisc.uib.es>

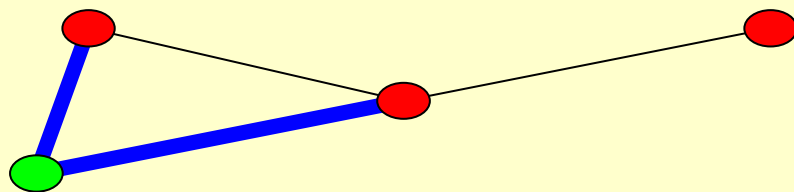
# Voter model

- “Voters” located in the nodes of a network have “opinions”  $\sigma_i=1$  or  $\sigma_i=-1$ .
- **A randomly chosen voter takes the opinion of one of its neighbors (node update).**



**Qs?: When and how one of the two absorbing states (**consensus**) is reached? Effect of network of interactions?**

Order Parameter: Average interface density



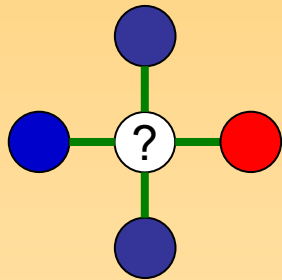
$$\rho = \frac{1}{2N\langle k \rangle} \left( 1 - \sum_{i=1}^N \sum_{j \in \nu(i)} \sigma_i \sigma_j \right)$$

**$\rho=0$  in absorbing state**

Interface: a link connecting nodes with different states.

# MODELS of CONSENSUS with *TWO OPTIONS* :

- Prototype models *with excluding options*: - VOTER MODEL  
- SPIN FLIP KINETIC ISING MODEL T=0



$$p_{\text{?} \rightarrow B} = 3/4$$

$$p_{\text{?} \rightarrow A} = 1/4$$

$$p_{\text{?} \rightarrow B} = 1$$

$$p_{\text{?} \rightarrow A} = 0$$

Voter Model

**RANDOM IMITATION**

Spin Flip  
Kinetic Ising T=0

**SOCIAL PRESSURE**

- ⊙ Active
- Option A
- Option B



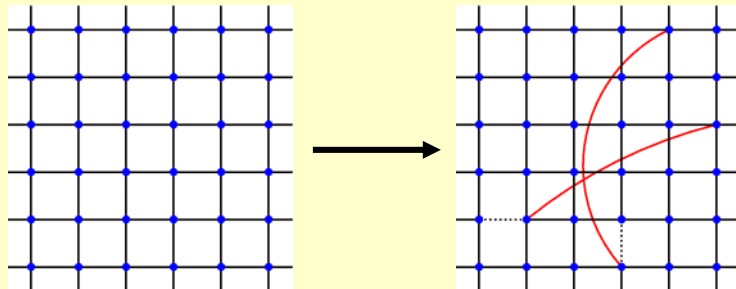
**IFISC**



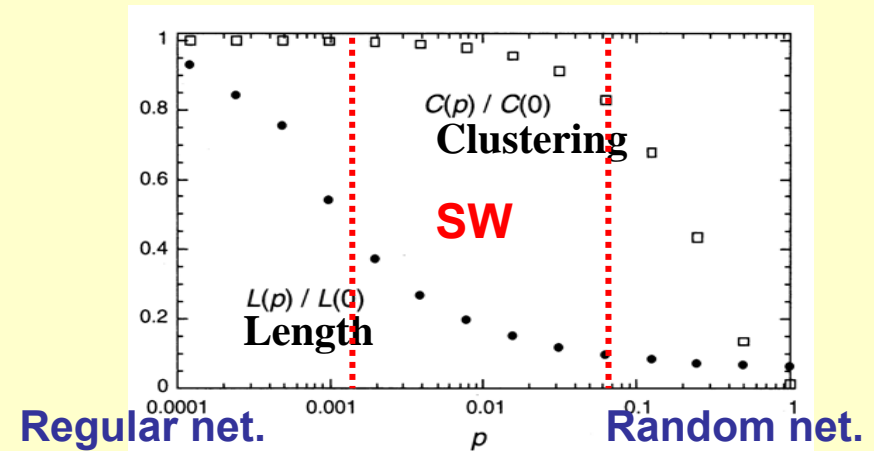
<http://ifisc.uib.es>

## Small World Networks

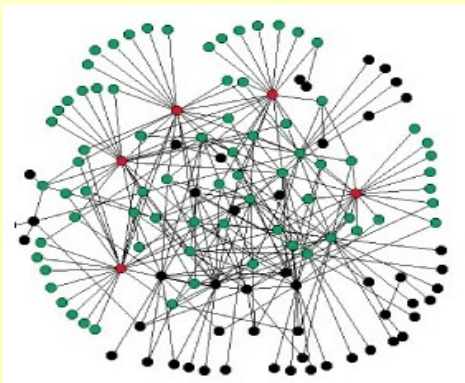
Watts-Strogatz, *Nature* **393**, 440 (1998)



Rewire with prob.  $p$



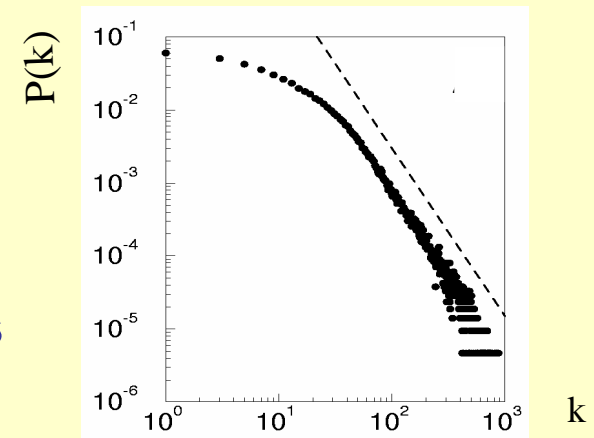
## Barabasi-Albert Scale Free networks



Power law for the degree distribution

$$P(k) \sim k^{-\gamma}, \gamma=3$$

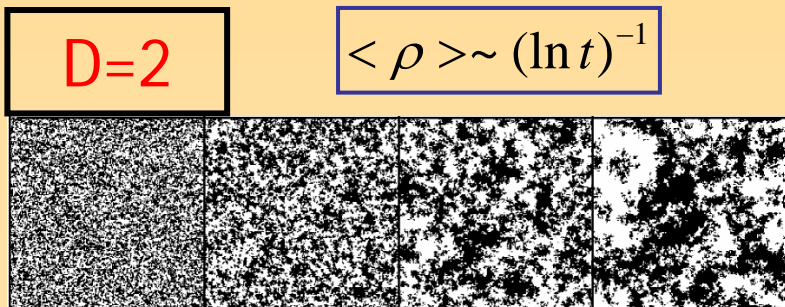
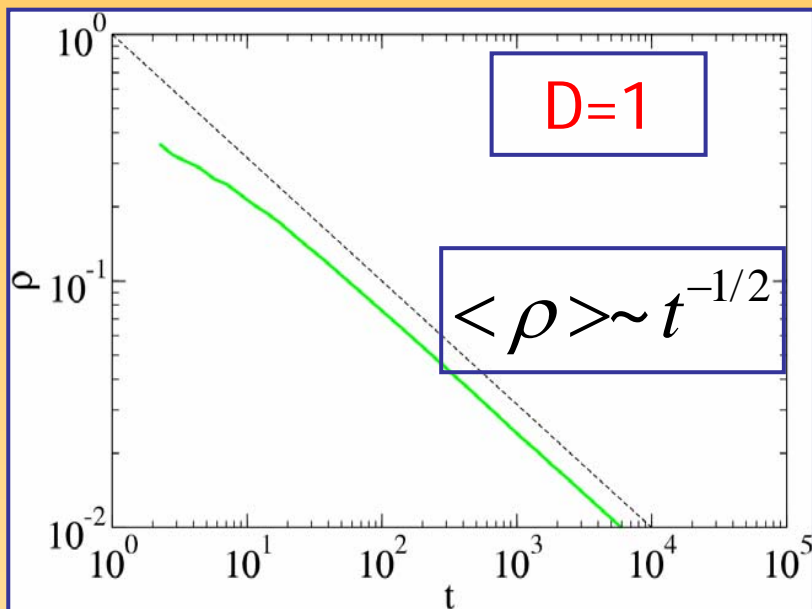
Importance of hubs



Albert & Barabasi, *Rev. Mod. Phys.* **74**, 47 (2002)

# Voter Model

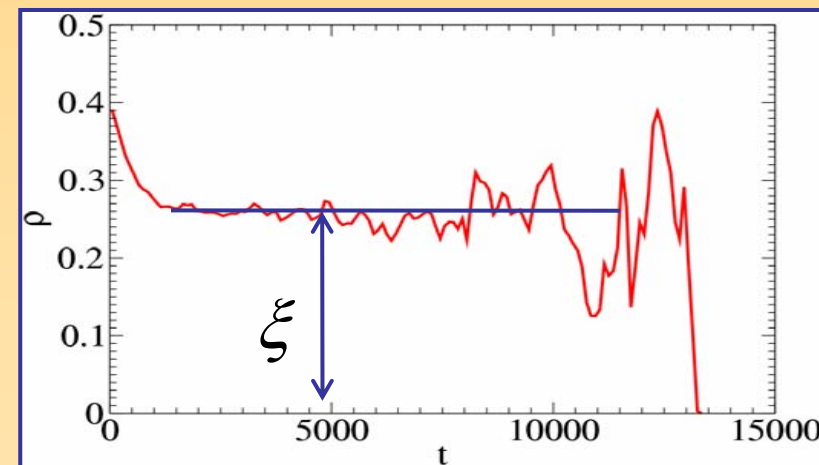
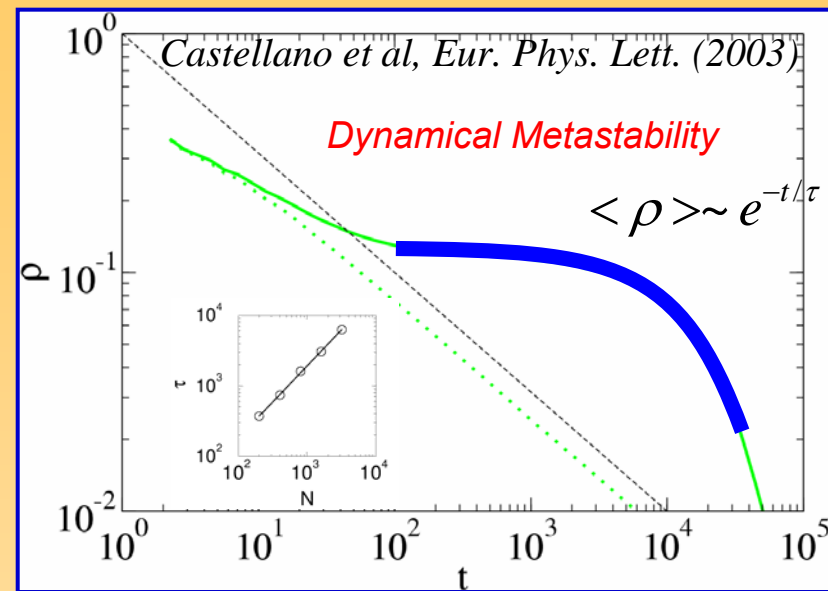
## Regular Networks



Coarsening without surface tension:  
Driven by interfacial noise

Dornic et al, Phys. Rev.Lett. (2001)

## Small World Networks



$l = \xi^{-1}$  Characteristic size of ordered domain

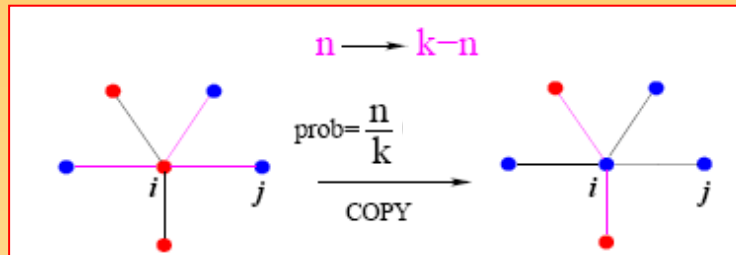
# Mean Field Voter model

F. Vázquez, V. M. Eguíluz and M. San Miguel (2007)

Mean Field Node Dynamics:  $\frac{d \langle \sigma \rangle}{dt} = 0$

## Mean Field Link Dynamics:

$\rho$  = global density of active links  
 $n$  = active links  
 $n-k$  = inert links  
 $k$  = degree of node  $i$



$$\Delta \rho = \frac{2(k - 2n)}{\langle k \rangle N}$$

Node  $i$  of degree  $k$ :  $\left. \frac{d\rho}{dt} \right|_k = \frac{1}{1/N} \sum_{n=0}^k B(n, k) \frac{n}{k} \frac{2(k - 2n)}{\langle k \rangle N}$

$B(n, k)$  = Prob. that node  $i$  has  $n$  active links

$B(n, k) \approx \frac{k!}{n!(k-n)!} \rho^n (1 - \rho)^{k-n}$  ← **Mean Field:**  $\rho \sim$  prob that a link from node  $i$  is active

$$\frac{d\rho}{dt} = \sum_k P_k \left. \frac{d\rho}{dt} \right|_k = \frac{2\rho}{\langle k \rangle} [(\langle k \rangle - 1)(1 - 2\rho) - 1]$$

➔

$$\rho^s = \frac{\langle k \rangle - 2}{2(\langle k \rangle - 1)}$$

# Voter model in Random Networks

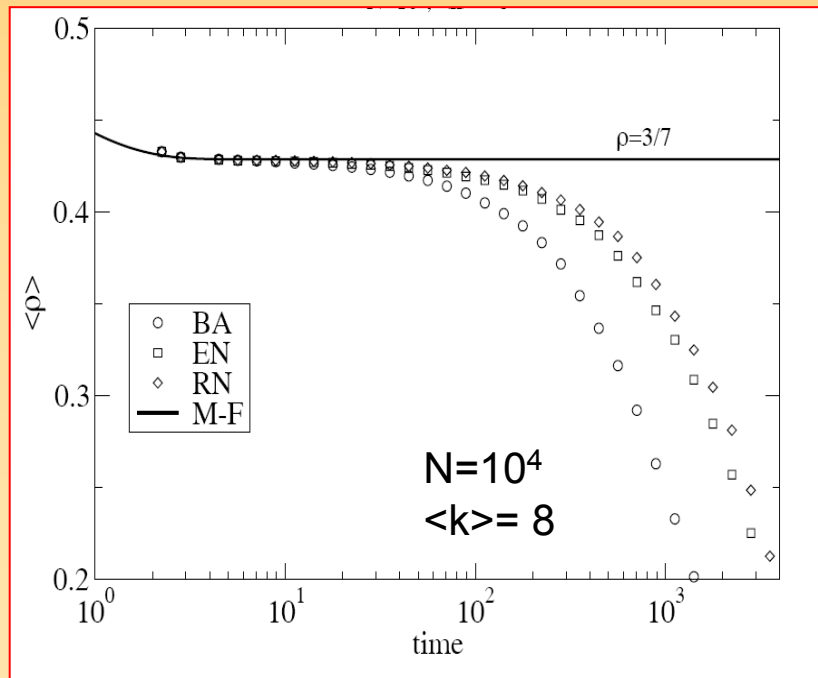
F. Vázquez, V. M. Eguíluz and M. San Miguel (2007)

## Mean Field Link Dynamics:

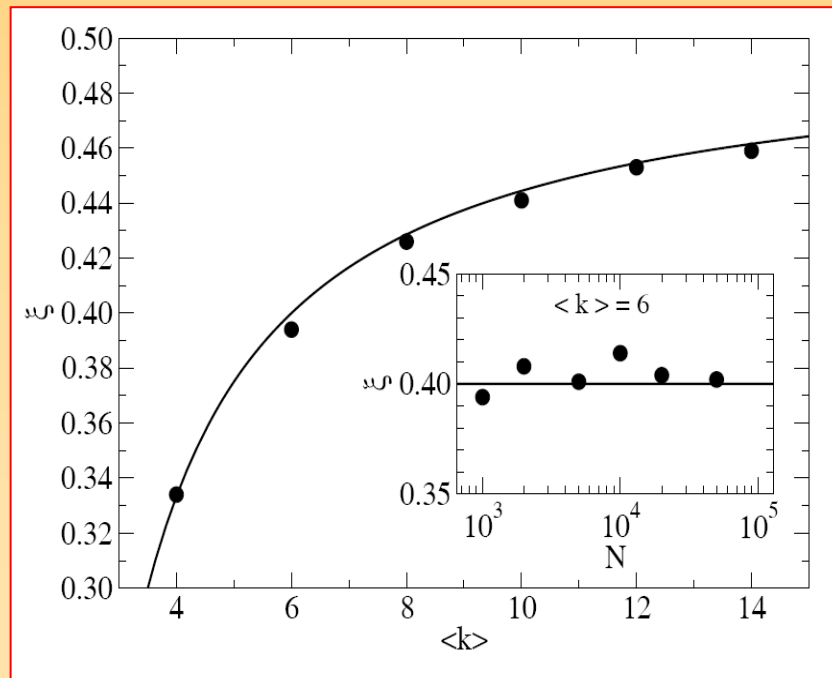
Single parameter theory

$$\rho^s = \xi = \frac{\langle k \rangle - 2}{2(\langle k \rangle - 1)}$$

## Network topology independence

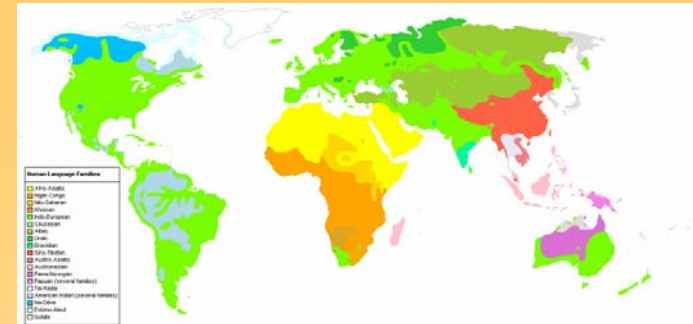


## Barabasi-Albert Scale Free Networks

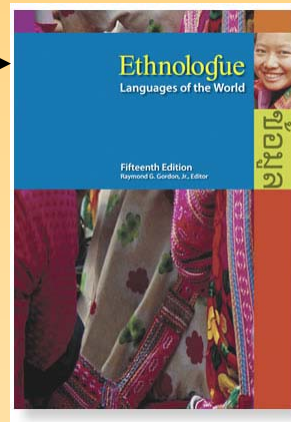


# Languages today

- There exist around 6000 languages in the world.
- Over 50% of them are endangered (UNESCO).
- 4% of languages account for 96% of people.
- 25% of languages have less than 1000 speakers.



◆ <http://www.ethnologue.com/> →

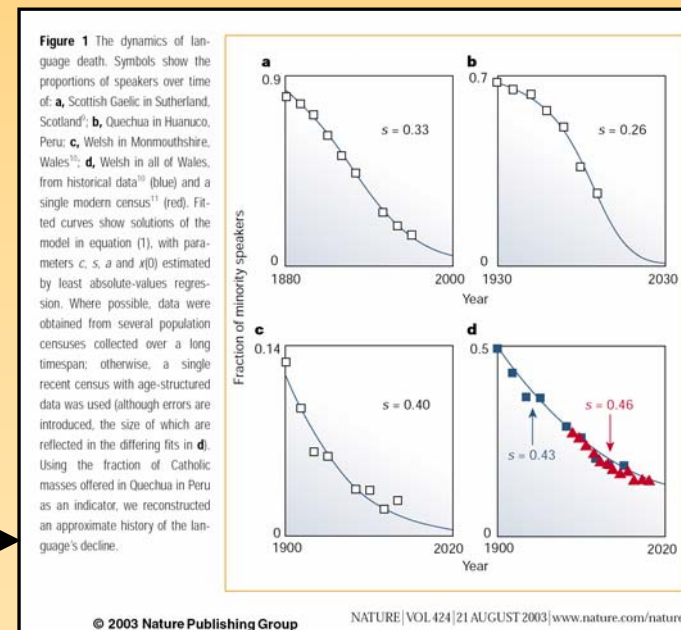


◆ <http://portal.unesco.org/culture>

◆ D. Crystal. *Language Death* (Cambridge CUP 2000)

Abrams, Strogatz (2003). *Nature* **424**, 900.

*“Question: Extinction of endangered languages”*



<http://ifisc.uib.es>

## Agent-Based ABRAMS-STROGATZ model.

*D. Stauffer, X. Castelló, V. M. Eguíluz and M. San Miguel, Physica A (2007)*

N agents within a network: nodes → agents  
links → social interaction

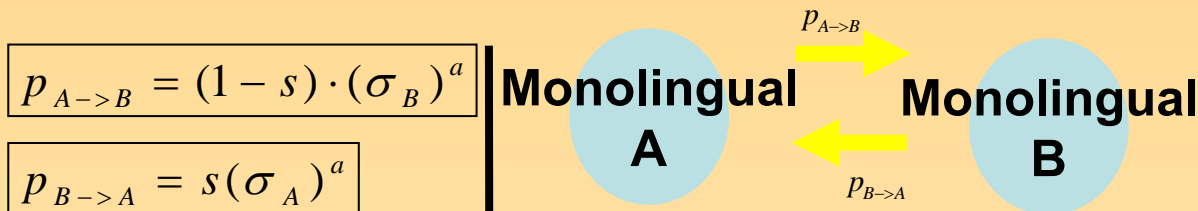
- States of the agents:
  - using language A: belonging to a monolingual community A.
  - using language B: belonging to a monolingual community B.

- Local density of speakers:

$$\sigma_{iA} = \frac{\# \text{ neighbours in state A}}{k_i}$$

$$\sigma_{iB} = \frac{\# \text{ neighbours in state B}}{k_i}$$

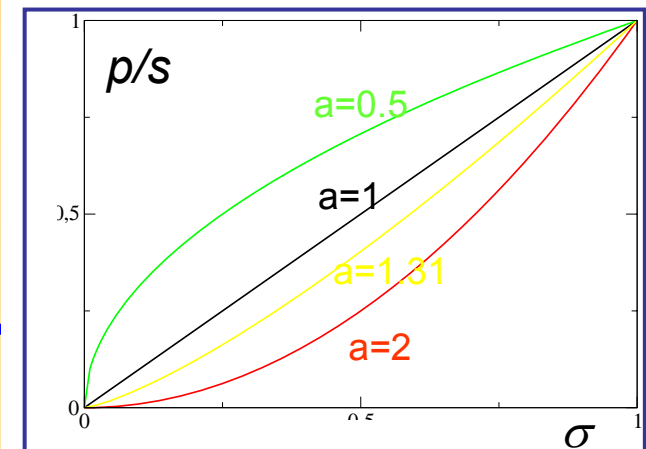
- Dynamics of interaction: choose randomly an agent,



**s: prestige of language A (s<sub>B</sub>=1-s)**

**a: exponent → determines the shape of p<sub>A→B</sub>**

**a > 1: Dominance      a < 1: Coexistence**



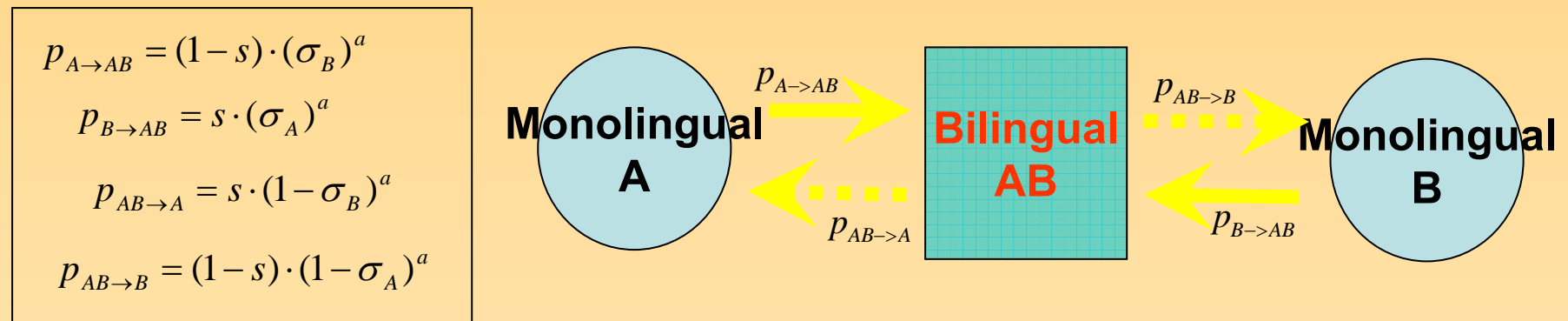
s=0.5, a=1

Dynamics of voter model

## MINETT-WANG: Agent-Based BILINGUALS MODEL.

N agents within a network: nodes  $\rightarrow$  agents  
links  $\rightarrow$  social interaction

- States of the agents:
  - using language A: belonging to a monolingual community A.
  - using language B: belonging to a monolingual community B
  - using both, A & B: belonging to the bilingual community AB.
- Dynamics of interaction: choose randomly an agent,



- Order parameter : *interface density*:

$$\rho = \frac{\# \text{ links joining different states}}{\text{total \# of links}}$$

## MEAN-FIELD APPROACH. Socially equivalent languages

### ABRAMS-STROGATZ

$$\left\{ \begin{array}{l} \frac{d\sigma_A}{dt} = 0 \\ \sigma_B(t) = 1 - \sigma_A(t) \end{array} \right.$$

Any proportion of A-speakers is a marginally stable solution:

*The average density of A-speakers does not change with time*

### BILINGUALS MODEL

$$\left\{ \begin{array}{l} \frac{d\sigma_A}{dt} = \frac{1}{2} \left[ (1 - \sigma_A + (\sigma_B)^2 - 2\sigma_B) \right] \\ \frac{d\sigma_B}{dt} = \frac{1}{2} \left[ (1 - \sigma_B + (\sigma_A)^2 - 2\sigma_A) \right] \\ \sigma_{AB}(t) = 1 - \sigma_A(t) - \sigma_B(t) \end{array} \right.$$

There exist 3 fixed points:

► two stable, corresponding to the dominance of one of the languages:

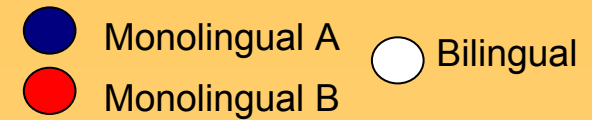
$$\begin{aligned} (\sigma_A, \sigma_B, \sigma_{AB}) &= (1, 0, 0) \\ &= (0, 1, 0) \end{aligned}$$

► and one unstable (unstable coexistence)

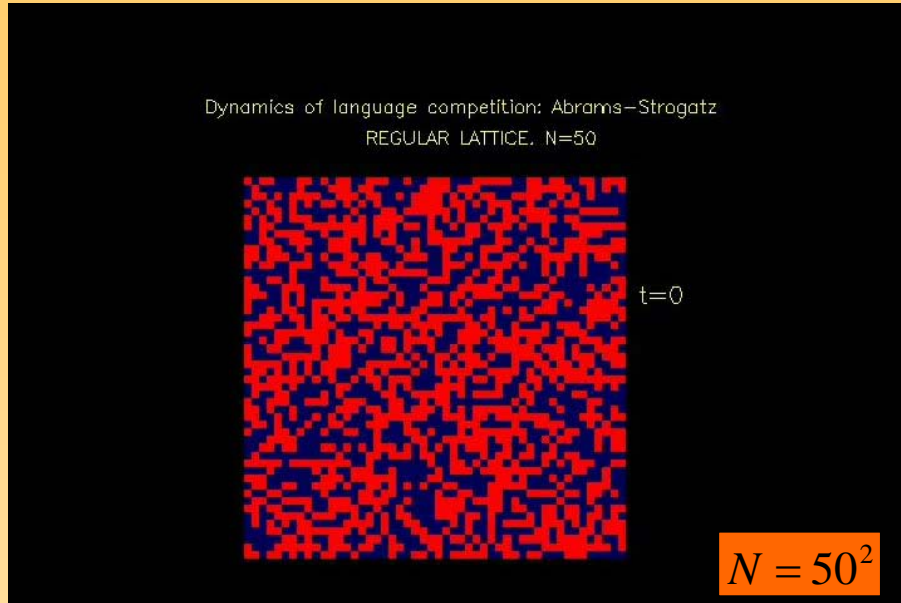
$$(\sigma_A^*, \sigma_B^*, \sigma_{AB}^*)$$



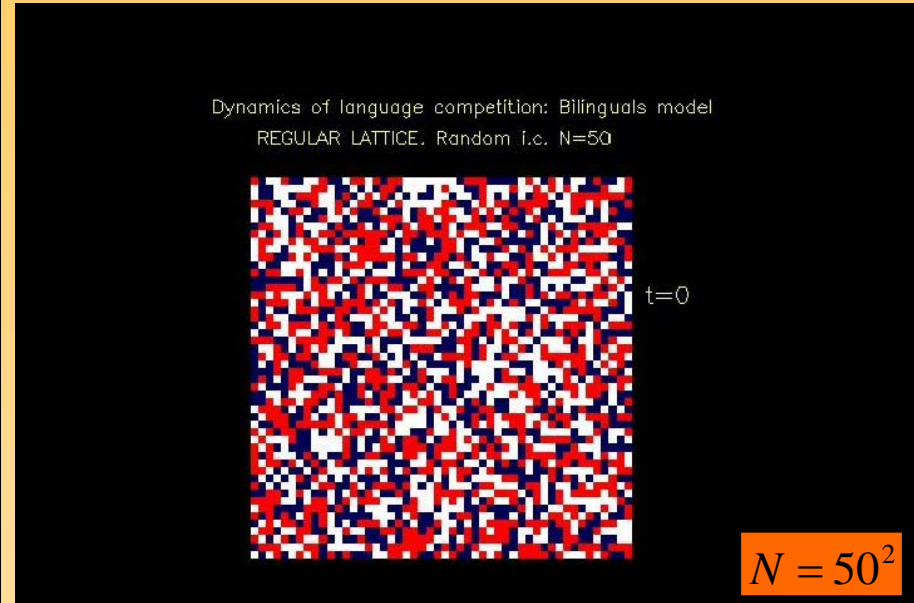
# LOCAL EFFECTS: Regular network



## ABRAMS-STROGATZ



## BILINGUALS MODEL



### Coarsening: Growth of monolingual spatial domains

► Dynamics: Interfacial noise

► NO bilingual domains. Bilinguals are **interfaces**

► Dynamics: Curvature driven

*Castelló et al., New J. Phys. 8, 308 (2006); in Advancing Social Simulation, S. Takahashi et al eds, Springer (2007)*



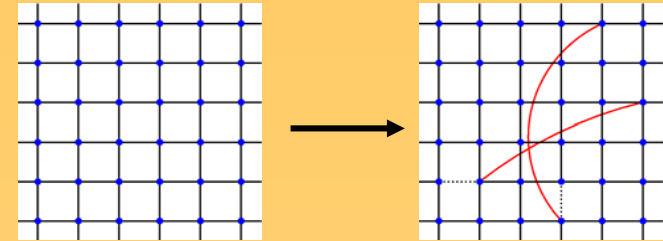
**IFISC**



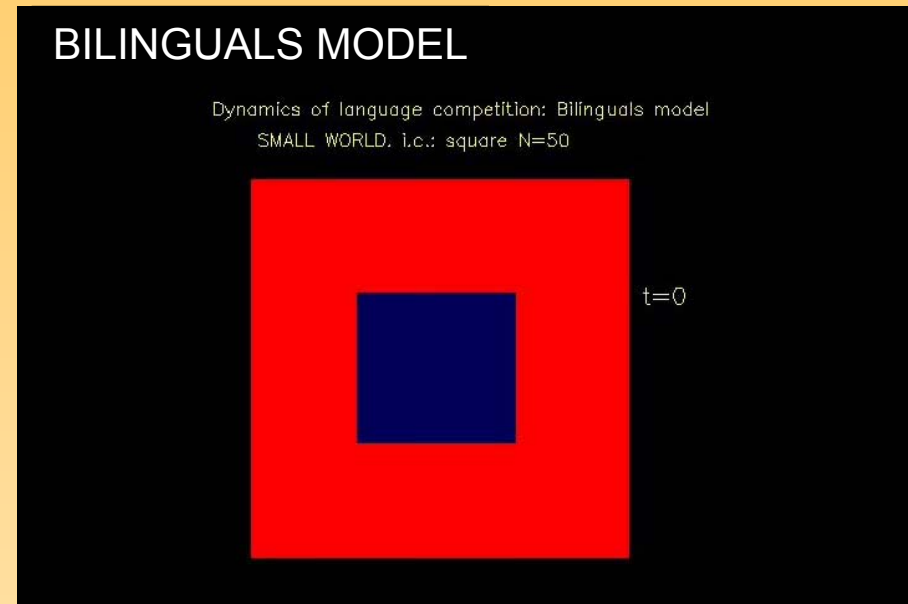
<http://ifisc.uib.es>

# Social Structure: Small World net

● Monolingual A ● Monolingual B ○ Bilingual



Rewire with prob.  $p$



$p = 0.1 \quad N = 50^2$

**Bilingual agents + Small World produce fast extinction of one monolingual community**

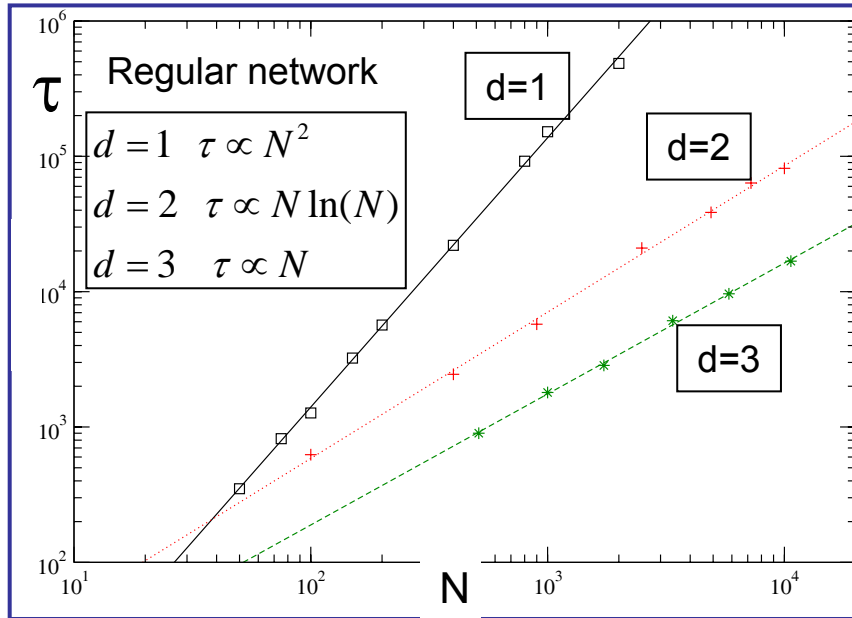
*Castelló et al., New J. Phys. 8, 308 (2006); in Advancing Social Simulation, S. Takahashi et al eds, Springer (2007)*



<http://ifisc.uib.es>

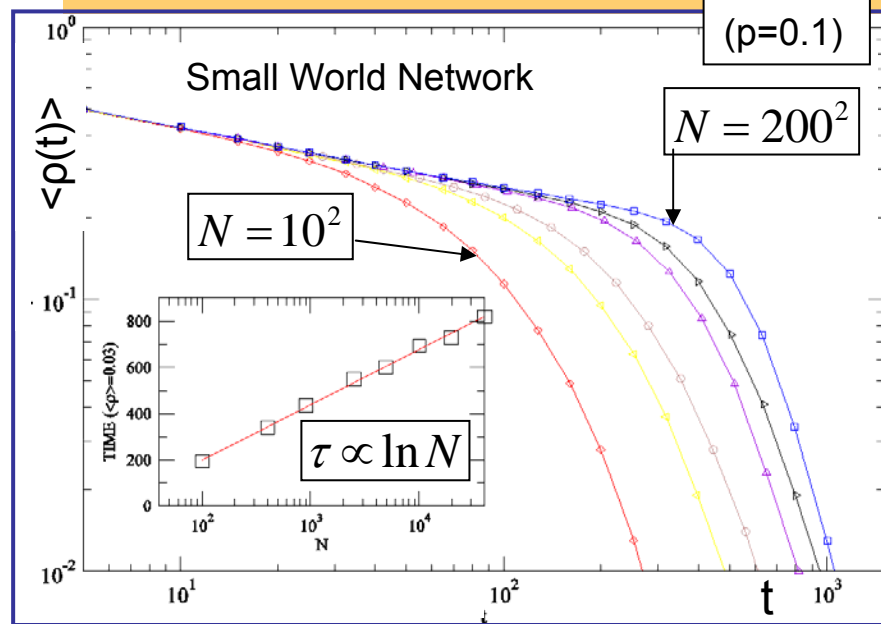
# Time to extinction: Dependence on system size.

ABRAMS-STROGATZ



- Small World net:  $\tau \propto N$
- Results consistent with *voter model*

BILINGUALS MODEL



- 2-d regular net:  $\tau \propto N^2$



In a SW network bilingual agents cause a faster extinction:  $\tau_{BM} / \tau_{AS} \propto \ln N / N$

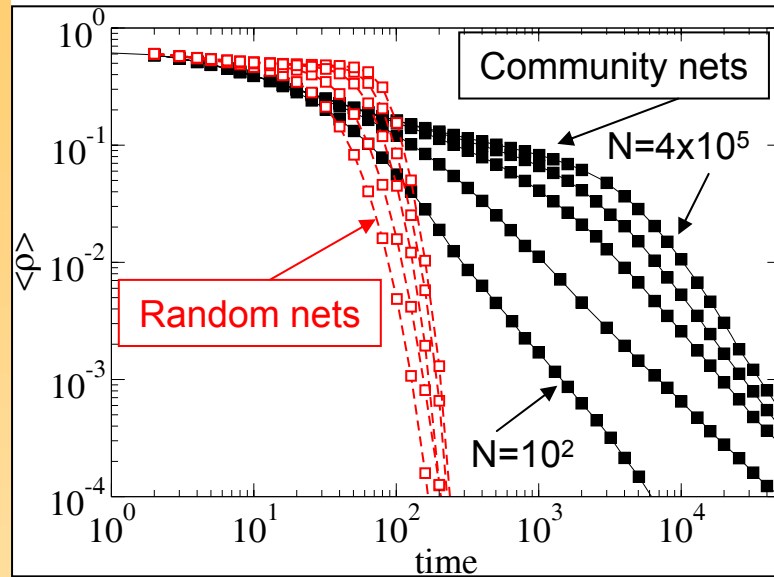
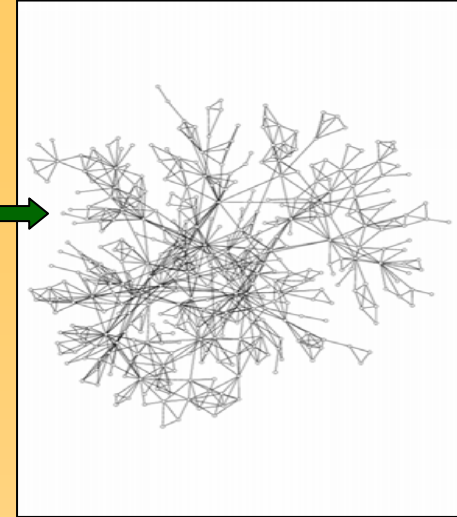
Castelló et al., *New J. Phys.* **8**, 308 (2006); in *Advancing Social Simulation*, S. Takahashi et al eds, Springer (2007)



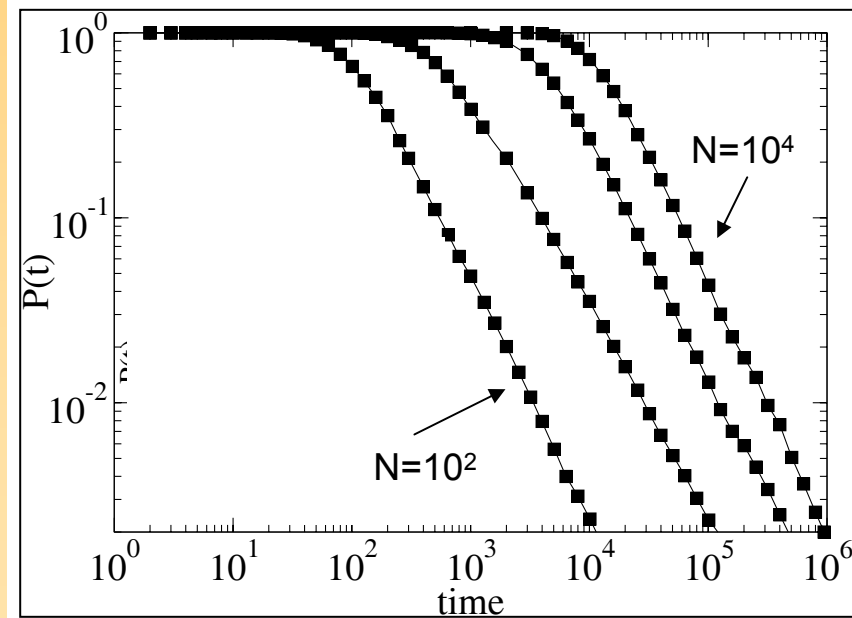
<http://ifisc.uib.es>

# BILINGUALS MODEL IN NETWORKS WITH COMMUNITY STRUCTURE

R. Toivonen, et al *Physica A* 371, 851 (2006)



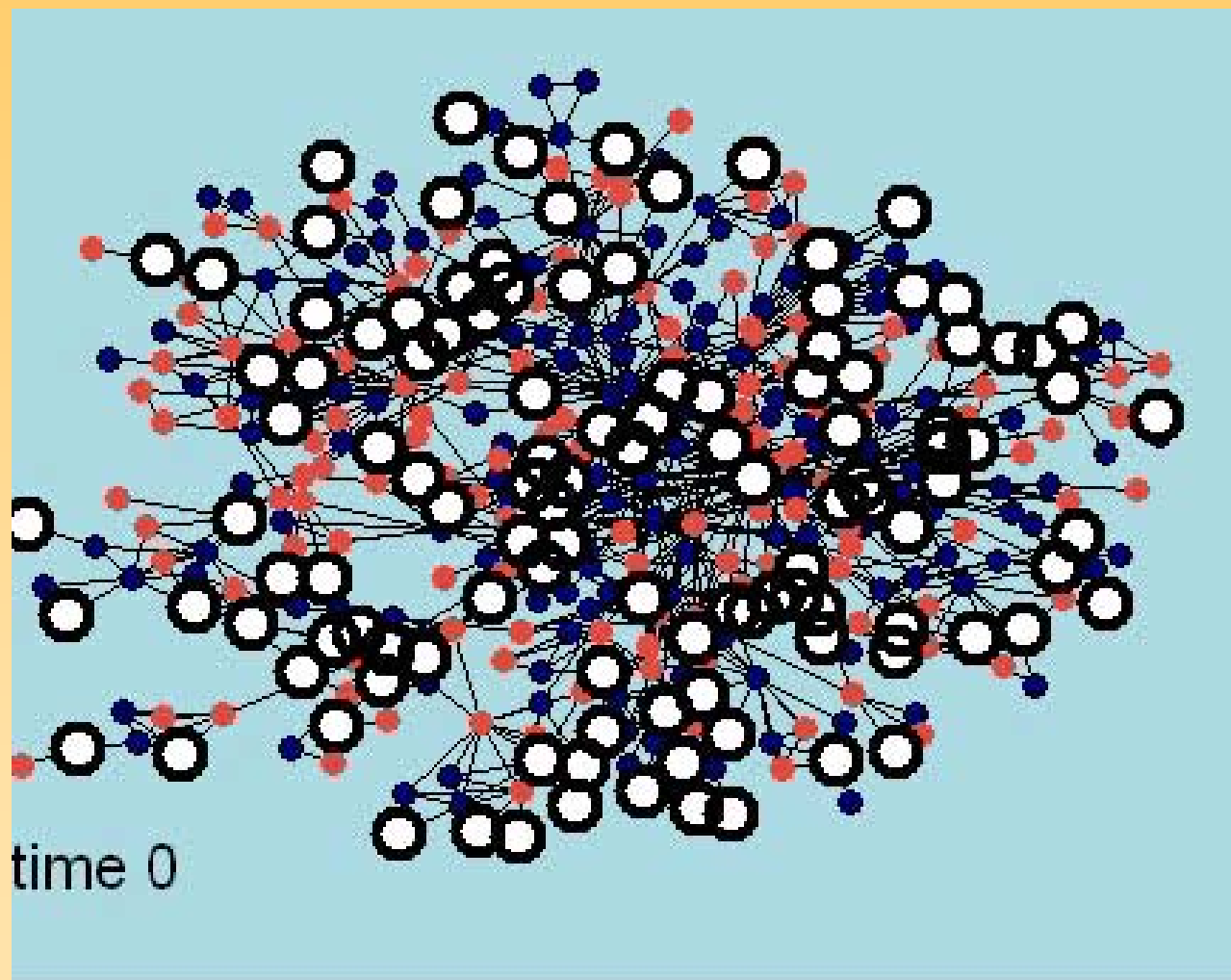
Power law decay for the distribution



► Community structure as a mechanism of resilience

There is no characteristic time scale !! → alive runs at any time scale

## Bilinguals model in CommNet: Time evolution



### Legend

Monolingual A



Monolingual B



Bilingual AB



## Closer inspection: Analysis of single runs

### CLASSES OF REALIZATIONS:

(i)

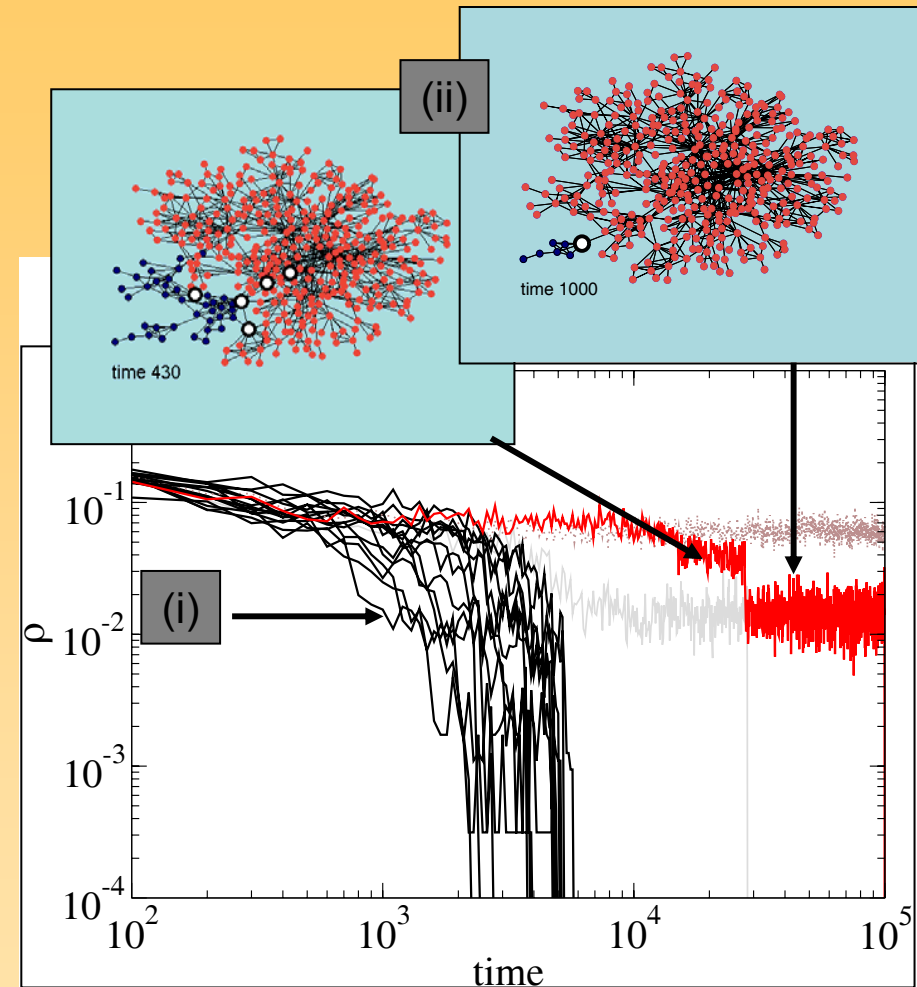
decay to absorbing state after coarsening

(ii)

fall into long-lived **trapped metastable states**

Several plateaus →

**Metastable states at all time scales**

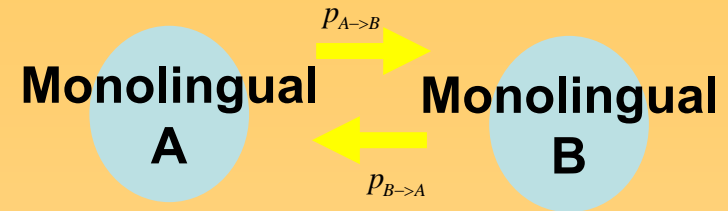


# Parameters in the Dynamics of Language Competition

ABRAMS-STROGATZ

$$p_{A \rightarrow B} = (1 - s) \cdot (\sigma_B)^a$$

$$p_{B \rightarrow A} = s (\sigma_A)^a$$



$s$ : Social **PRESTIGE** (language property)

$s > 0.5$  Preference for language A;  $s = 0.5$  Socially equivalent languages

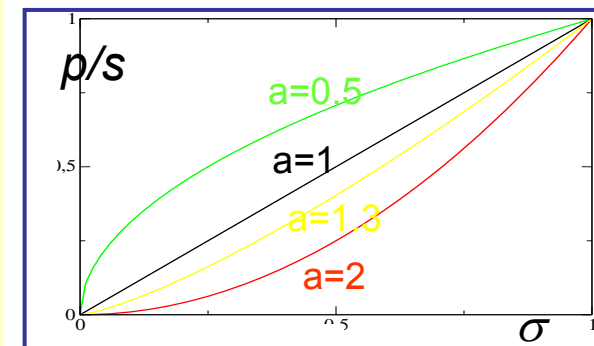
$a$ : Language **VOLATILITY** or persistence parameter (social dynamics property)

Measures volatility of language state as a nonlinear effect

$a = 1$  Marginal situation: Random imitation process

$a > 1$  Low volatility: Probability below random imitation

$a < 1$  High volatility: Probability above random imitation



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# Parameters in the Dynamics of Language Competition

## 1) Marginal **VOLATILITY** $a=1$

*Prestige dominates. Bilinguals not important when there is preferred language*



## 2) High **VOLATILITY** $a<1$ ( $a=0.2$ )

i) *Social equivalent languages ( $s=0.5$ ):*

- Long lived coexistence
- No well defined linguistic borders
- Bilinguals not important



ii) *Effect of prestige ( $s=0.6$ ):*

- Long lived coexistence without bilinguals
- Bilingual agents cause fast extinction of weak language

## 3) Low **VOLATILITY** $a>1$ ( $a=3$ )

i) *Social equivalent languages ( $s=0.5$ ):*

- Coarsening + well defined linguistic borders
- Bilinguals not important

ii) *Effect of prestige ( $s=0.6$ ):*

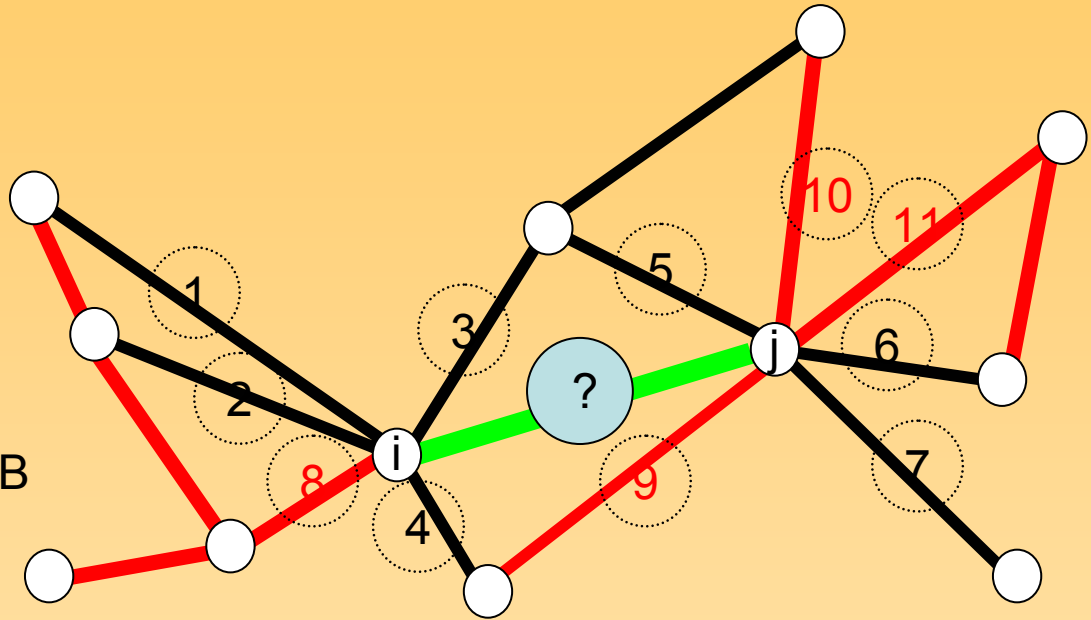
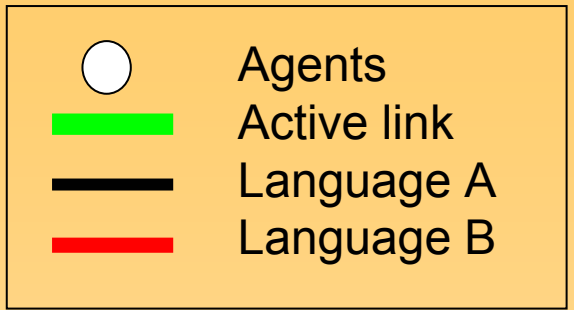
- Dominates causing extinction of weak language
- Bilingual agents play no important role



**Bilinguals:**

- Define linguistic borders for marginal volatility and no prestige
- Cause extinction for high volatility and different prestige

# LINK Dynamics: Language is a link, not an agent property



$\sigma_m$  proportion of m-links of contacts from i and j. m=A, B

$$p_{? \rightarrow B} = \sigma_B = 4/11$$

$$p_{? \rightarrow A} = \sigma_A = 7/11$$

$$a=1, s=0.5$$

Types of agents

Monolingual A	100% A-links
Monolingual B	100% B-links
Bilingual agent	q % A-links 100-q % B-links

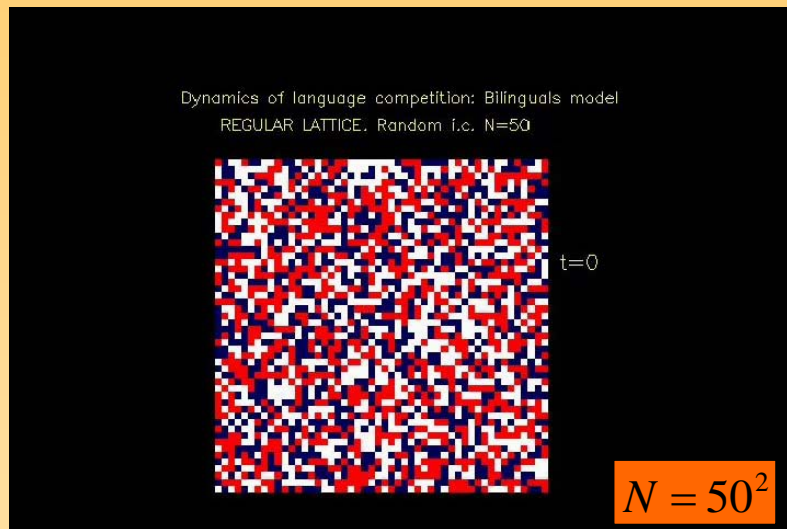
↳ different degrees of bilingualism

# Bilinguals: Agents vs. Link Dynamics

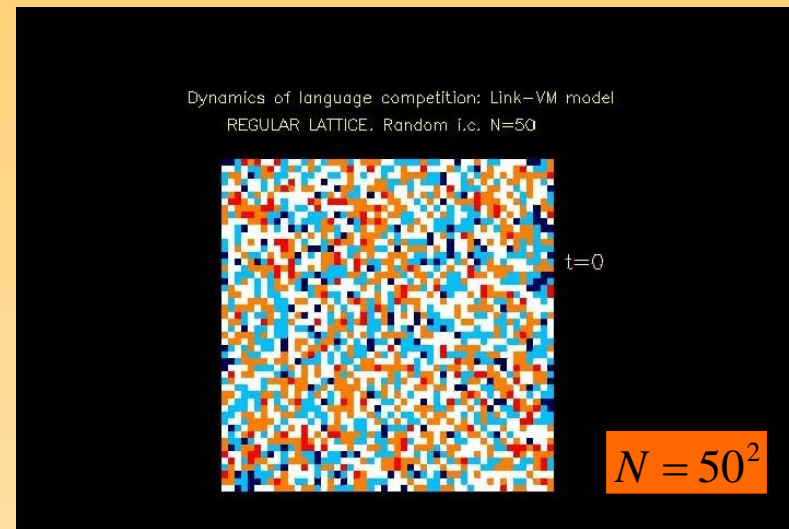
Regular network, marginal volatility ( $a=1$ ), no prestige ( $s=0.5$ )



Agent-based model



Link-based model



**Coarsening: Growth of monolingual spatial domains. No bilingual domains**

- ▶ Bilinguals are the **interfaces** (linguistic borders)
- ▶ Dynamics: Curvature driven
- ▶ **FASTER** growth of the monolingual domains.

- ▶ Broad linguistic borders
- ▶ Dynamics: **Interfacial noise**
- ▶ **SLOWER** growth of the monolingual domains.



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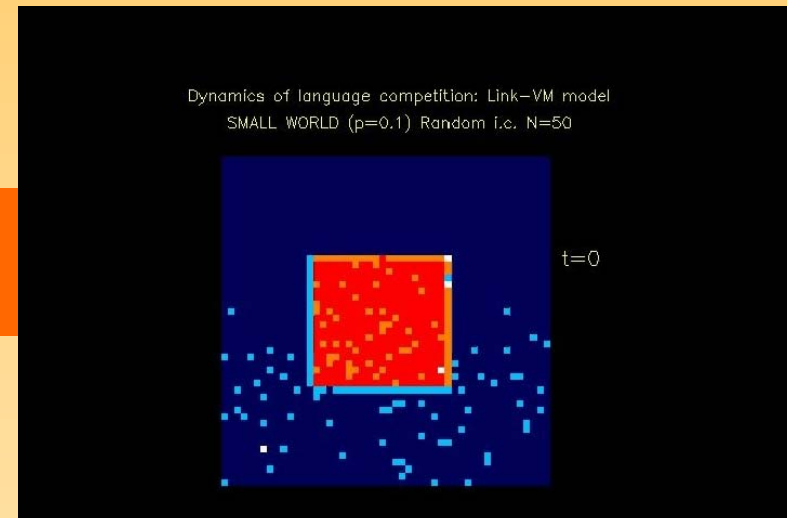
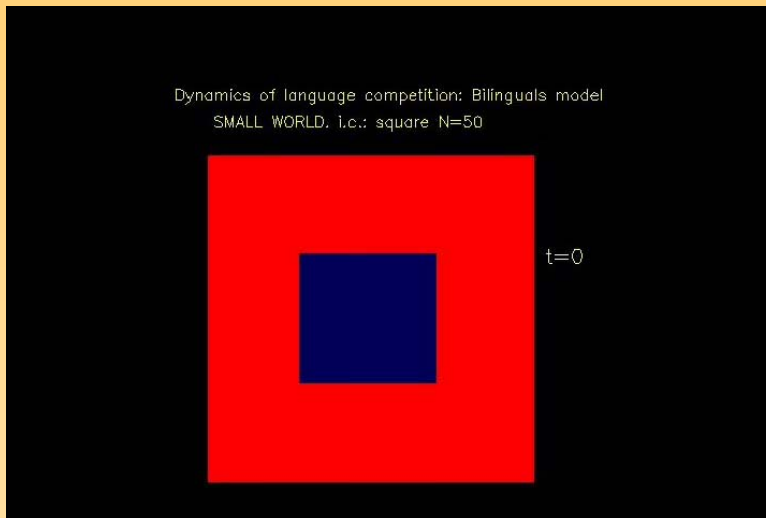
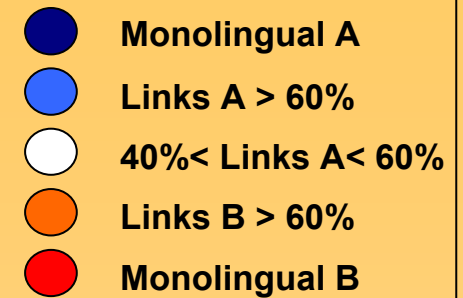
# Bilinguals: Agents vs. Link Dynamics

## Small World Network

marginal volatility ( $a=1$ ), no prestige ( $s=0.5$ )

Agent-based model

Link-based model



$p = 0.1$   
 $N = 50^2$

► *Fast extinction of one monolingual community*

► *Long lived metastable states of language coexistence*  
► *Large fraction of bilingual agents in state of coexistence ~ 40%*



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# OPEN QUESTIONS and FURTHER RESEARCH.

## ■ Different fundamental dynamics:

- **CODE-SWITCHING:** Language evolution to new AB language (eg. *Spanglish* )
- Resilience vs. cross-breeding: *World standard spoken English*
- From imitation or majority dynamics to cost/benefit optimization

## ■ Further analysis of present models:

- Distinguish between **knowledge/competence** and **social use** of a language
- Language use depending on social environment. **DIGLOSSIA**
- Language difference: Spanish-Catalan vs. French-Flemish
- Political measures: Competition of agent-agent interaction and agent-field.  
Field: external, endogenous, local, global...
- Coupling to **cultural diffusion models** where language is a prominent cultural feature
- More **complete models to define language**: bit string models,  
...(use of grammar universals: syntactic order,...)
- Data and case study? Validation



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