Coordinating/searching/learning on volatile networks: simple models

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References:
MM, F. Slanina, F.Vega-Redondo, Proc. Nat. Acad. Sci. 2004
G.C.M. Ehrhardt, MM, F.Vega-Redondo Phys. Rev.E 2006, Int. J.Game Theory 2006
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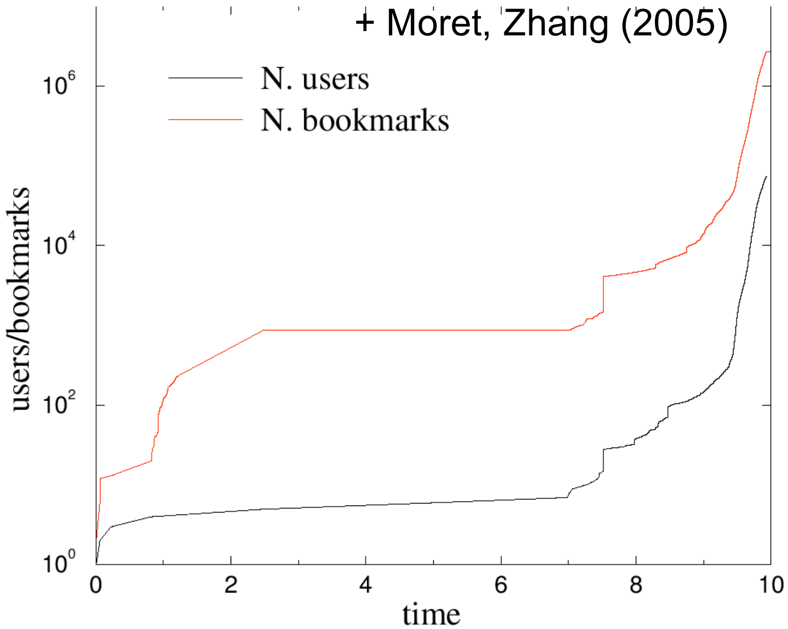
An example: Information sharing/gathering systems and web communities (e.g. del.icio.us)



Under what conditions do such networks emerge?
What is the value of this system? (e.g. yahoo bought flickr 15M$)
Will it continue grow in this way?

We live in network society/economy (Castells 2001)

- Labor markets (Granovetter, Montgomery, Calvó-A., Jackson)
- R&D partnerships (Gulati et al, Goyal & Joshi)
- Scientific collaboration (Newman, Goyal et al)
- Organizational performance (Radner, Garicano, Cabrales et al)
- Social bargaining (Corominas-Bosch, Calvó-Armengol, Polanski)
- Interindustrial trade (Kranton & Minehart)
- International trade (Rauch, Greif)

And in the past?

What is a link?

Links are the **backbone of economic interaction** but they also have other functions (social capital) such as **carrying information, enforcing coordination, promoting similarity,** etc

A potential interaction (e.g. a collaboration) may require:

- Information on existence/reputation of potential partners (often provided by common neighbors) → proximity
- Coordination (e.g. protocols, technological standards, language) or synchronization
- Proximity or similarity in social, political, technological … dimension

Changing environment, volatility → links decay (cost to maintain link)

Social capital (=set of links) acts as a buffer against volatility

Red Queen effect: “It needs all the running you can do to keep in the same place”

Main message

Under somewhat broad conditions

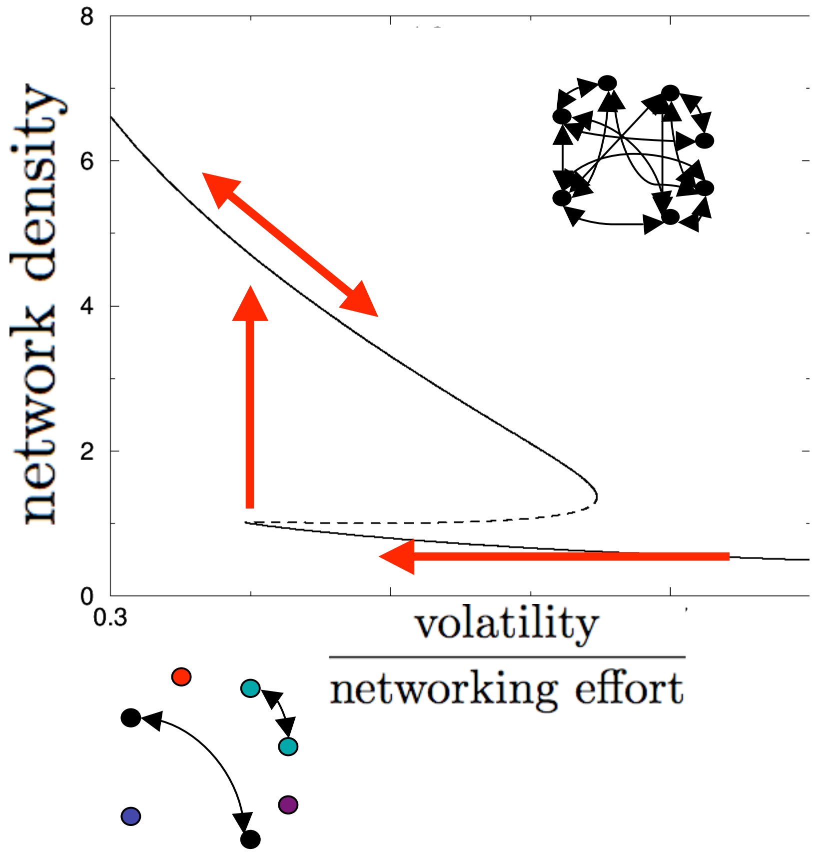
Sharp transitions: socio-economic networks are expected to emerge in an abrupt manner as a consequence of the feedback between networking efforts of individuals and the benefits the network provides in terms of coordination, information and innovation diffusion, social cohesion, …

Resilience: once dense networks form, they are robust to deterioration of external conditions

Coexistence: for the same environmental parameters, the network can either be dense or very sparse, depending on the history

More precisely

- Sharp transition
- Resilience
- Phase coexistence



Relevance?

- Understanding social-capital
  (e.g. what is the worth of del.icio.us)

- “Engineering” societies?
  (e.g. Silicon Valley, institution’s reforms)

- Sharp transitions as a basis for classifying stages of civilizations?

- …

Plan:

- Insight from a simple model:
  coordinating in a volatile network
  - Definition
  - The stationary state of a finite society
  - The dynamics of an infinite society
- A broader context
  - Innovation/opinion diffusion
  - Searching partners
- Conclusions

Coordinating on a network

- A society of N agents

  Each agent chooses a norm $s_i=1,\ldots,q$

- Norm revision

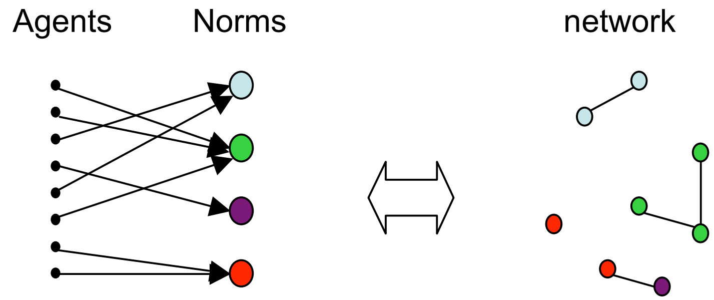
  At rate ν each agent i revises his/her norm and sets it to that which is most popular among his/her friends (with random tie breaking)

- Link formation

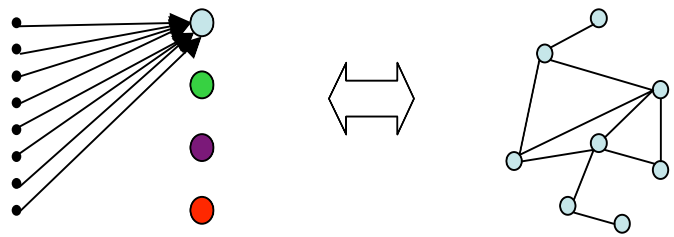
  At a rate η agent i meets an agent j drawn at random. If $s_i=s_j$ they enstablish a collaboration (i.e. a link).

- Environment volatility

  A profitable cooperation may turn unprofitable: each link decay at a rate λ



disorder – low link density



order – high link density

The Master equation

$$\frac{\partial P(\omega,t)}{\partial t} = \sum_{\omega' \in \Omega} [P(\omega',t)W(\omega' \to \omega) - P(\omega,t)W(\omega \to \omega')]$$

- Microscopic state
  - Network + norms: $\omega = \{a_{i,j}, s_i\}$,
    $a_{i,j}$= 0 (no link i-j) or 1 (i-j linked)
    s_i=1,…,q
- Link creation
  - $\omega \rightarrow \omega' = \{\omega_{-i,j}, a_{i,j}=1\}$, $W[\omega \rightarrow \omega'] = 2\eta(1-a_{i,j})/(N-1)$
- Link removal
  - $\omega \rightarrow \omega' = \{\omega_{-i,j}, a_{i,j}=0\}$, $W[\omega \rightarrow \omega'] = \lambda a_{i,j}$
- Norm revision
  - $\omega \rightarrow \omega' = \{\omega_{-i}, r_i = r'\}$, $W[\omega \rightarrow \omega'] = \nu$, r' majority norm

The stationary state I $\quad \text{finite } N \quad t \to \infty$

- Let $\quad \Omega_{=}\{\omega \in \Omega : \; s_i = s_j \; \forall (i,j) : \; a_{i,j} = 1\}$
- All states in $\Omega_=$ are ergodic, all states in $\Omega/\Omega_=$ are transient
  - Proof:
    links between agents with different s are never created
    all states in $\Omega_=$ can be reached passing from the empty network
- The invariant measure is

$$P_s(\omega) = P_0 \begin{cases} \prod_{i<j} z^{a_{i,j}} & \omega \in \Omega_= \\ 0 & \omega \notin \Omega_= \end{cases} \qquad z = \frac{2\eta}{(N-1)\lambda}$$

  - Proof: detailed balance

$$P(\omega', t)W(\omega' \to \omega) = P(\omega, t)W(\omega \to \omega')$$

The stationary state II

- The distribution of the fraction n_s of agents with s_i=s is given by

$$P_s(n_1,\ldots,n_q) = P_0 e^{-Nf(n_1,\ldots,n_q)}, \qquad n_1+\ldots+n_q=1$$

- For N large, $\{n_s\}$ is a.s. given by the minima of

$$f(n_1,\ldots,n_q) = -\sum_s \left[n_s \log n_s - \frac{z}{2} n_s^2\right], \qquad z = 2\eta/\lambda$$

- The solution can be characterized by the number L_+ of $n_s=n_+$ where n_+ (n_-) is the largest (smallest) solution of

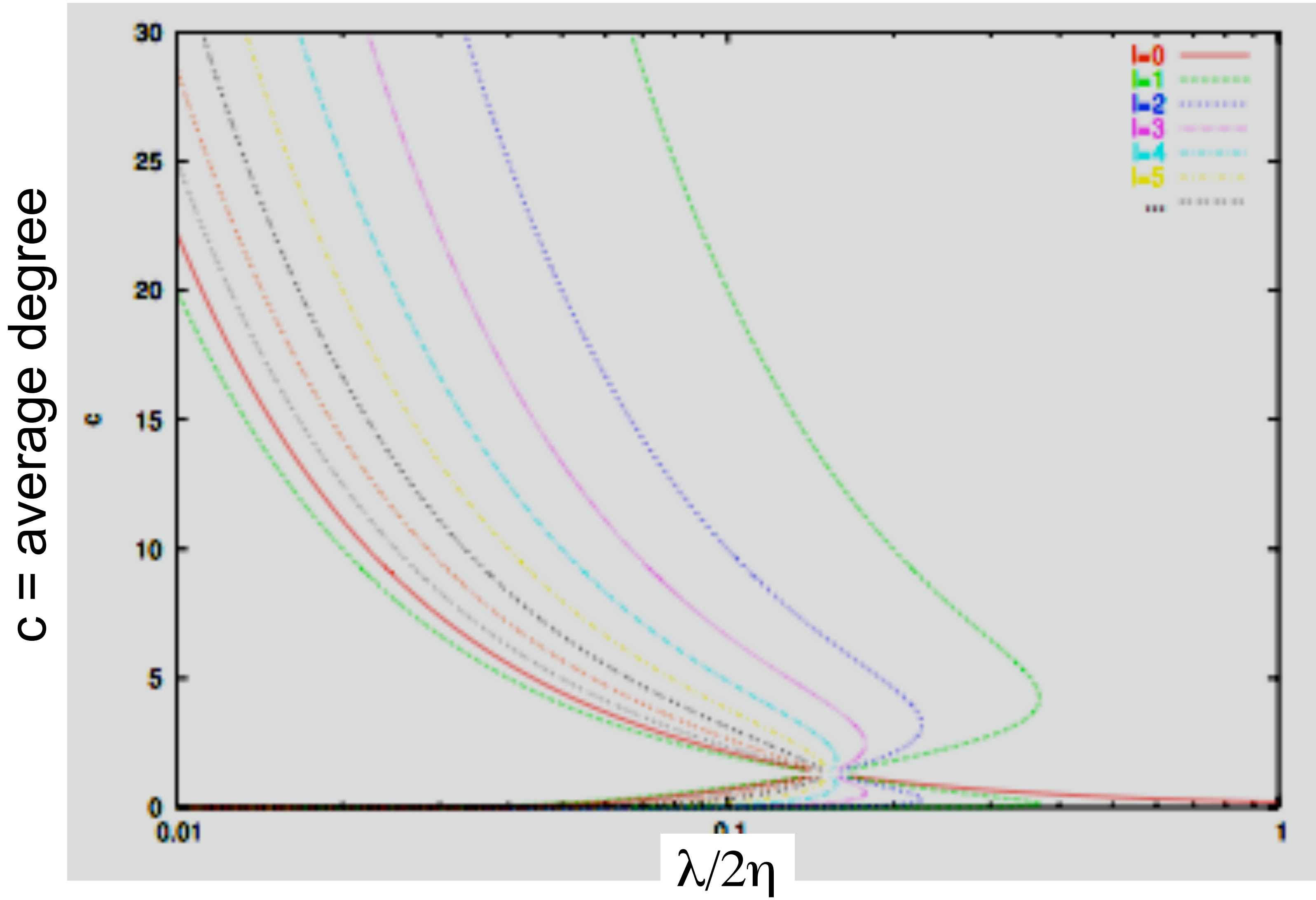
$$xe^{-zx} = \frac{n_0}{q}$$

n_0=fraction of isolated nodes (k=0)

- The L_+=0 solution exists and is a minimum for all z≤1
  L_+>1 solutions are saddle points
  L_+=1 solution is a minimum iif n_+ ➚ z

- In each component, the network is an Erdos-Renyi graph with $c=zn_s$

Stationary state III



The dynamics (t finite, N➔∞)

- Mean field dynamics

$$\begin{aligned} \dot{n}_{k,s} &= 2\eta n_s n_{k-1,s} + \lambda(k+1)n_{k+1,s} - 2\eta n_s n_{k,s} - \lambda k n_{k,s}, \qquad k > 0 \\ \dot{n}_{0,s} &= \lambda n_{1,s} - 2\eta n_s n_{0,s} + \nu \sum_r [n_{0,r} - n_{0,s}] \end{aligned}$$

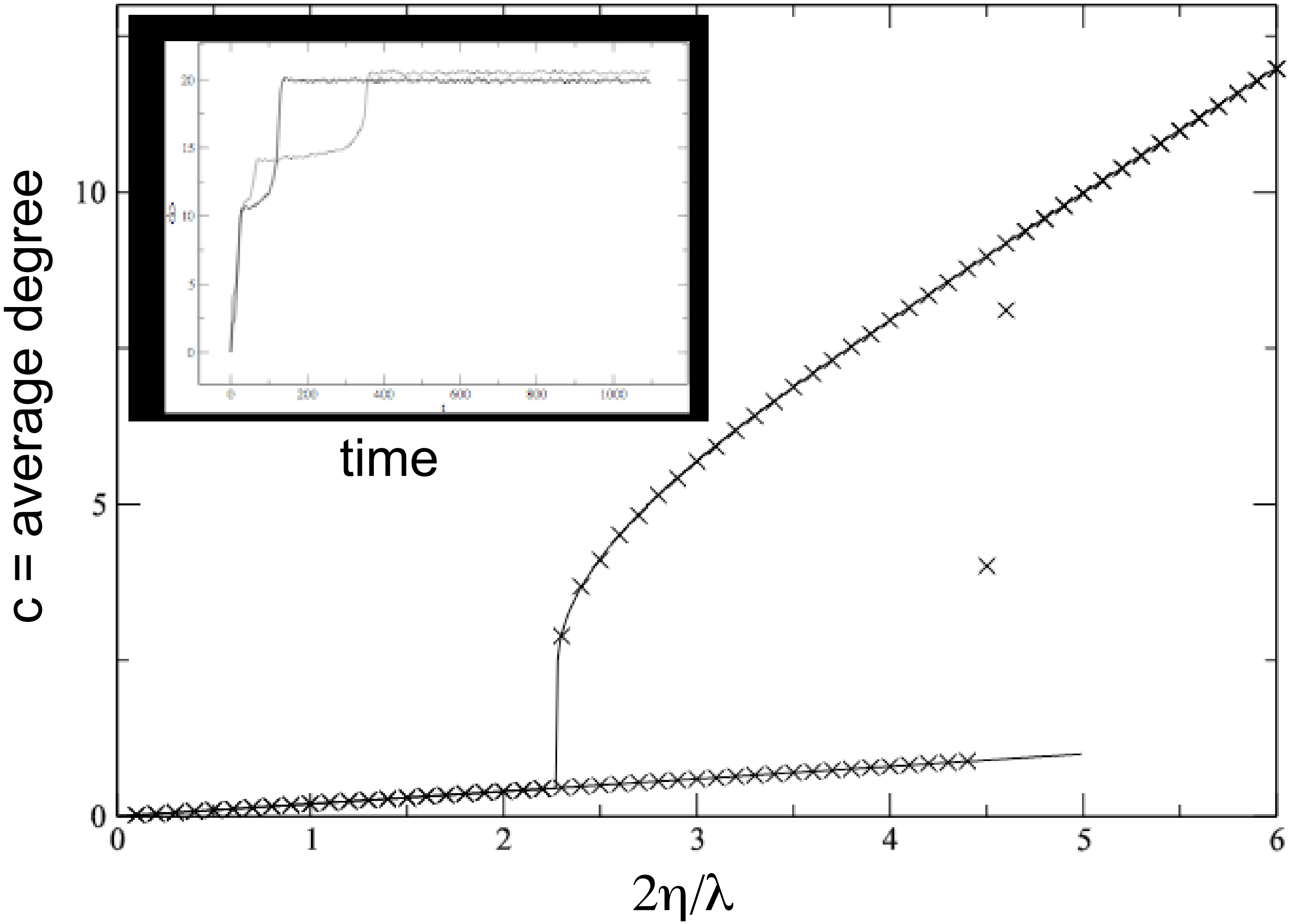
- If n_s➔n_s^* then

$$\lim_{t\to\infty} n_{k,s}(t) = \frac{(zn_s^*)^k}{k!} e^{-zn_s^*}$$

- The stationary states n_s^* are the same as those found above (min f ←➔ stability)
  - Proof: The Poisson transformation

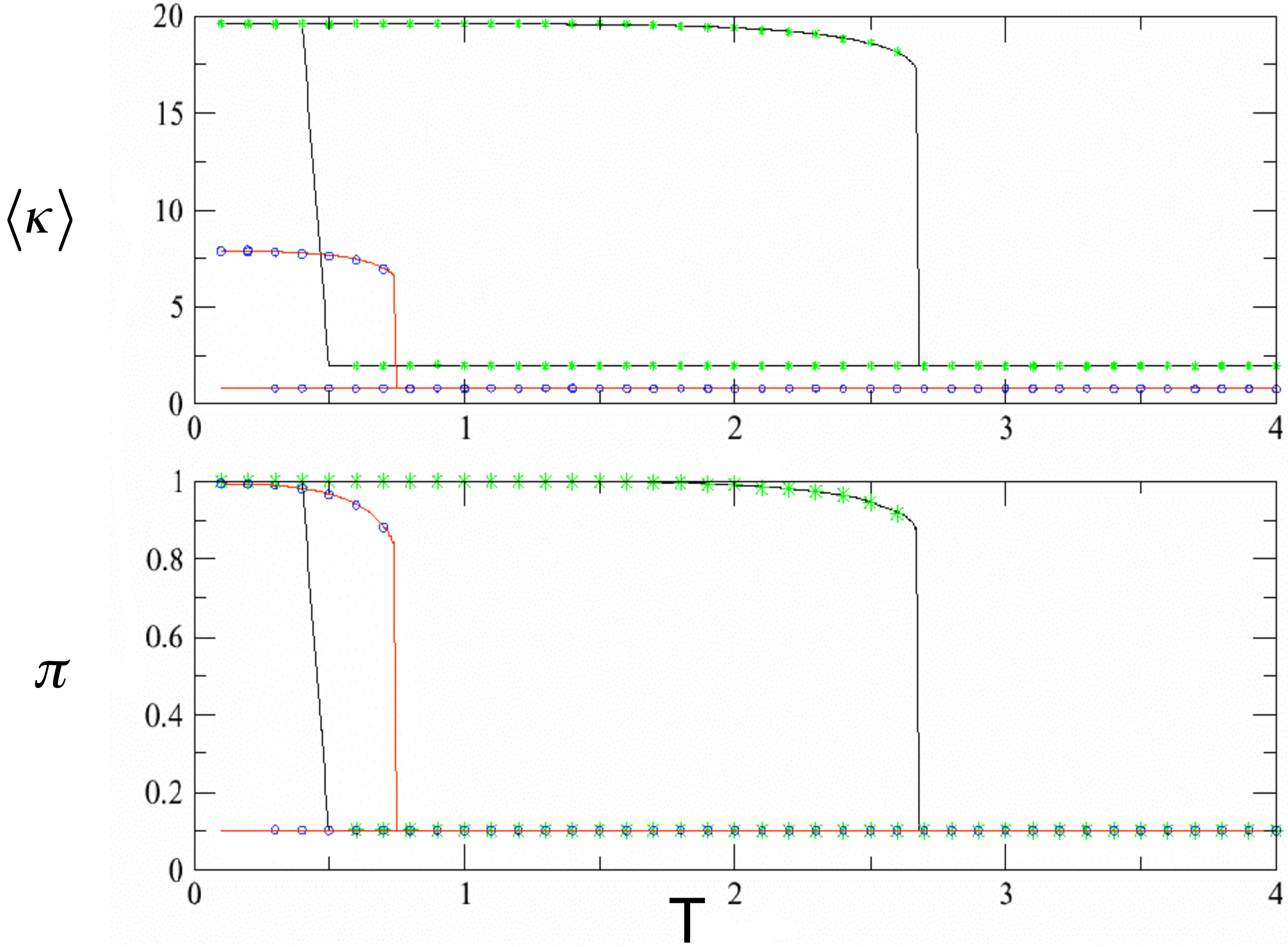
$$n_{k,s} = \int_0^\infty dx \frac{x^k}{k!} e^{-x} g_s(x,t), \qquad \Rightarrow \qquad \partial_t g_s = \lambda \partial_x (x - zn_s) g_s$$

Finite t and N: theory and simulations



Noisy choice

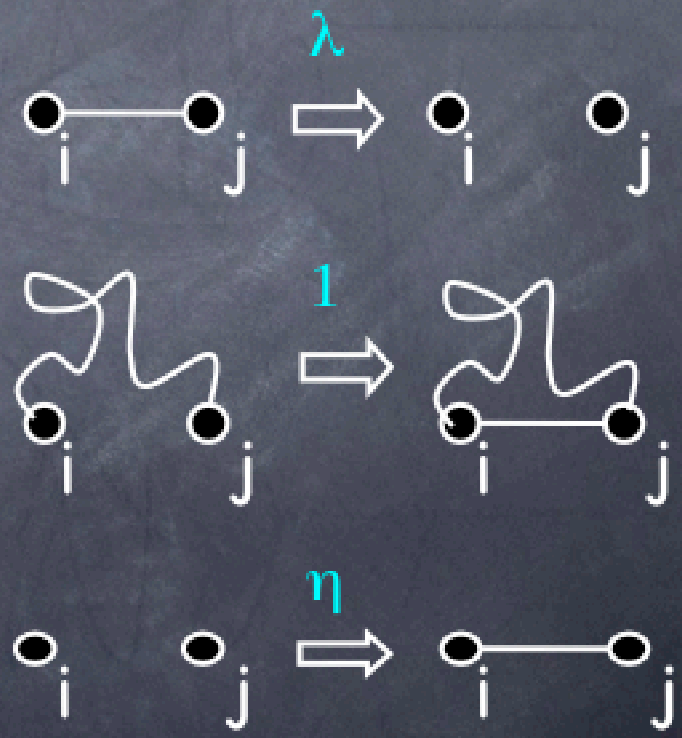
$P\{s_i=s\} \sim \exp\{k_{s,i}/T\}$, $k_{s,i}$= # friends of type s



(Potts model on a random graph)

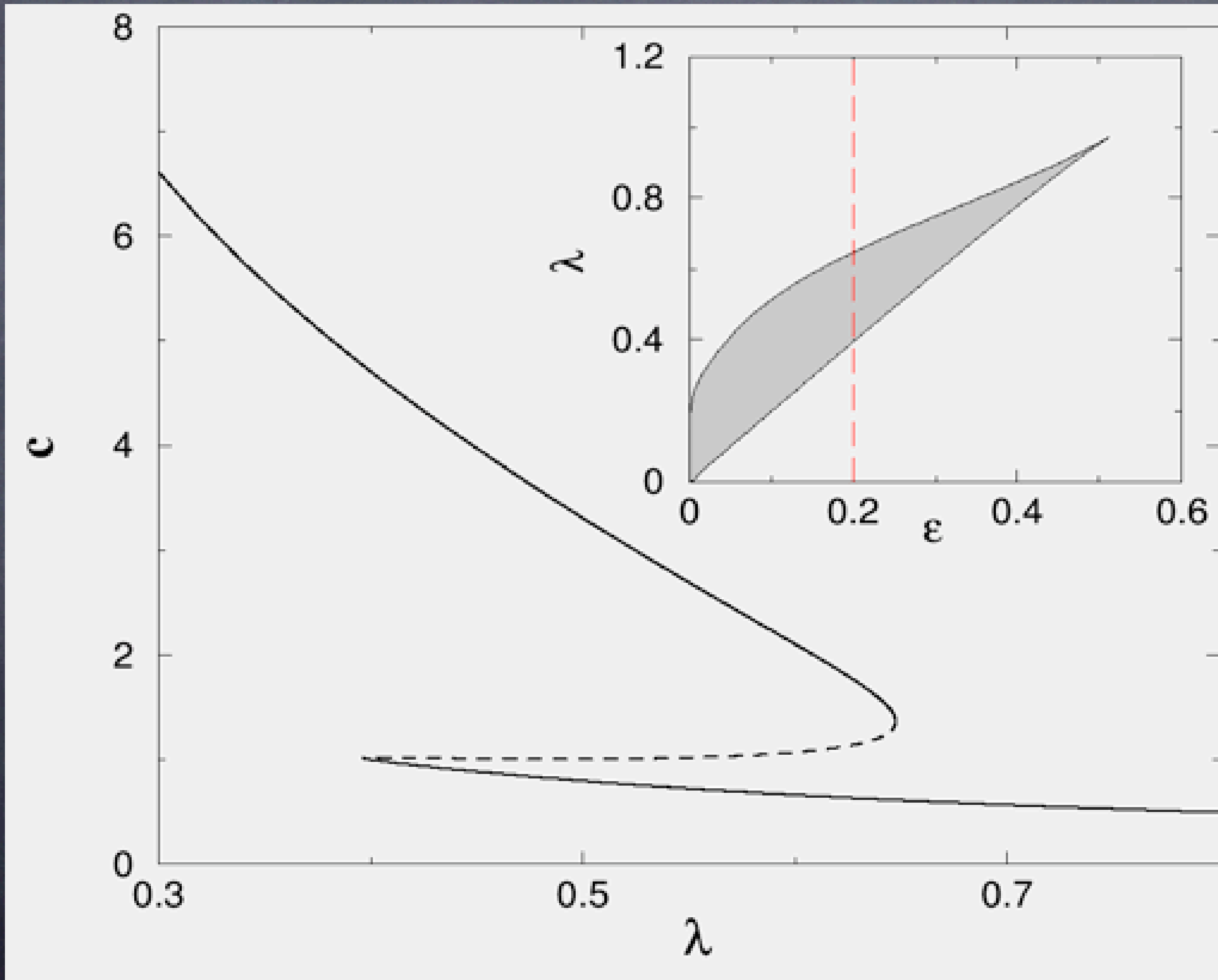
Clan-membership enhanced link formation

- A population of N agents in a network
- Each link decays/disappears at a rate λ
- At rate 1 *i* receives an opportunity to form a link with *j*. If there is a path on the network from *i* to *j* the link is formed, otherwise it is formed only with probability $\eta < \lambda$



Mean field theory

(random graph approximation)



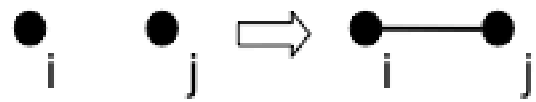
Knowledge/technology level $h_i(t)$

• linked agents tend to become similar

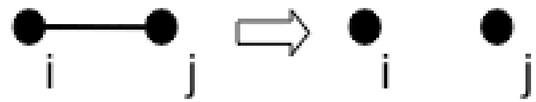
$$h_i(t) \to h_i(t^+) = \begin{cases} \max_{j \in N_i} h_j(t) & \text{techology adoption} \\ \frac{1}{|N_i|} \sum_{j \in N_i} h_j(t) & \text{knowledge diffusion} \end{cases}$$

• interaction is easier between similar nodes/agents

- Link formation at rate 1 if $|h_i - h_j| < \delta h$
  η otherwise

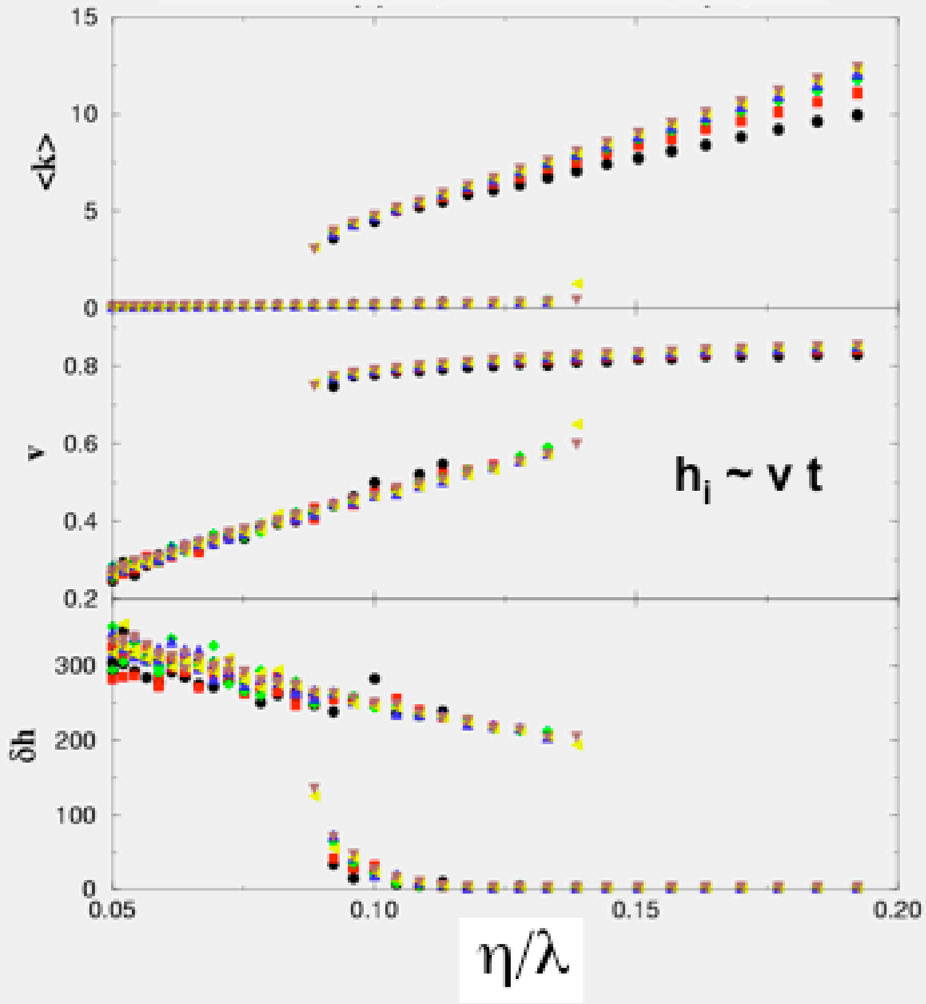


- Volatility λ



Technology adoption (KPZ growth)

- Spread of $h_i \downarrow c$
  $\rightarrow$ link formation rate $\uparrow c$
- Phase with slow growth, sparse network and large fluctuations of h
- Phase with fast growth, dense network and small fluctuations of h
- Sharp transition, coexistence and hysteresis



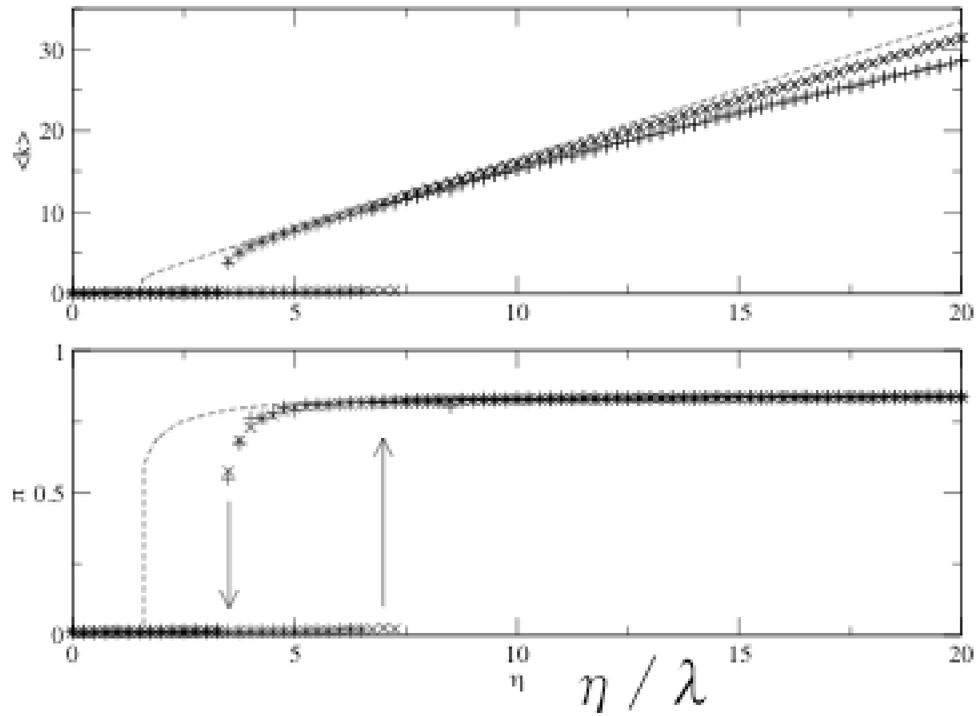
Knowledge diffusion

- Distribution of $h_i(t)$ from spectral density of Laplacian on random graphs (Dorogotsev et al., Rodgers & Bray, ...)

$$\langle (h_i - \langle h_i \rangle)^2 \rangle = \sum_{\mu>0} \frac{\nu\Delta}{2\mu} = \frac{\nu\Delta}{2} \int \frac{d\mu}{\mu} \rho(\mu)$$

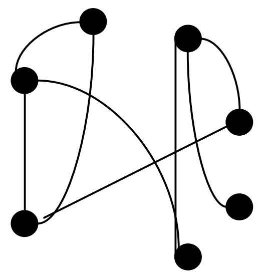
- average degree

- $P\{|h_i - h_j| < \delta h\}$



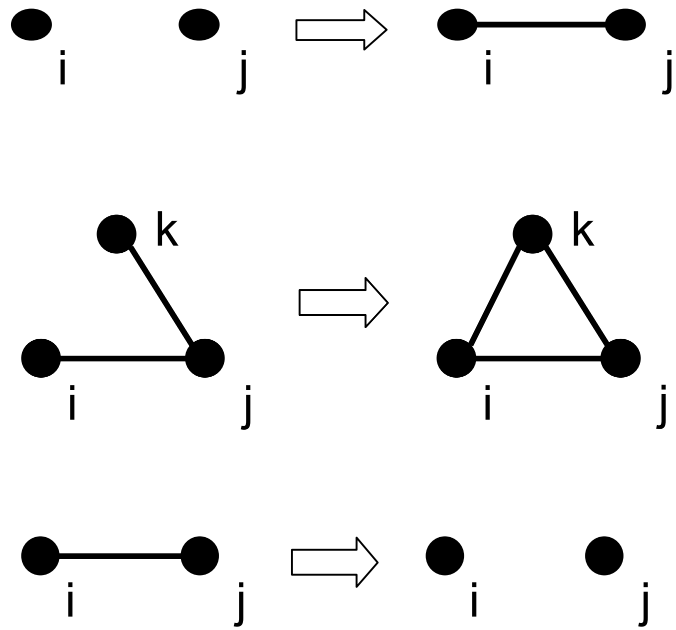
Searching partners in a volatile world

- A society of N agents
  Each pair of agents (i,j) can have a mutually profitable collaboration opportunity. In this case, if they get to know each other they form a link.



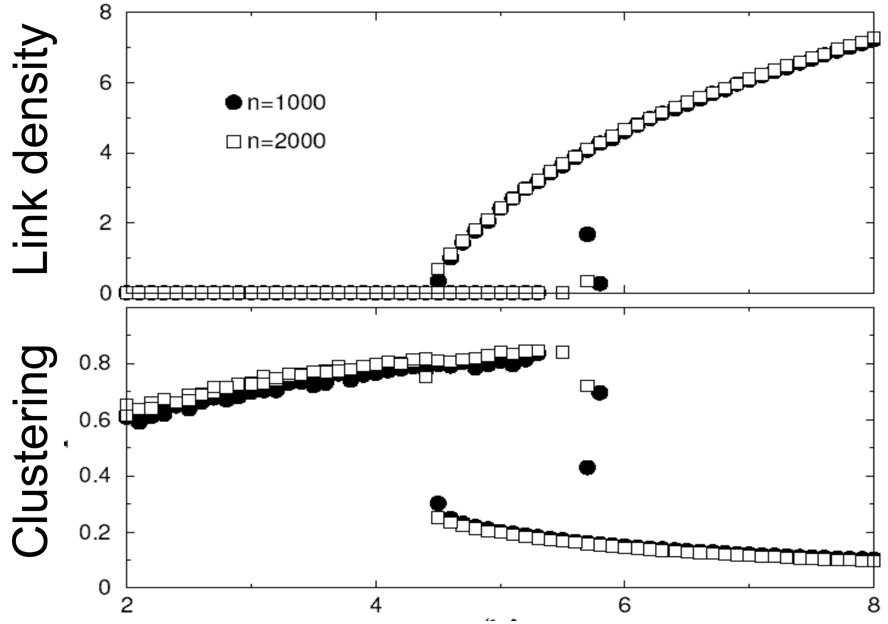
- Search for partners
  Global search: rarely (at a rate η) by random encounters:
  i meets j, randomly drawn from the population
  Local search: Typically (at a rate $\xi \gg \eta$) through neighbors:
  i ask at one of his friends* about one of his friends*

- Volatile environment
  A profitable cooperation may turn unprofitable with some rate λ – the corresponding link is deleted

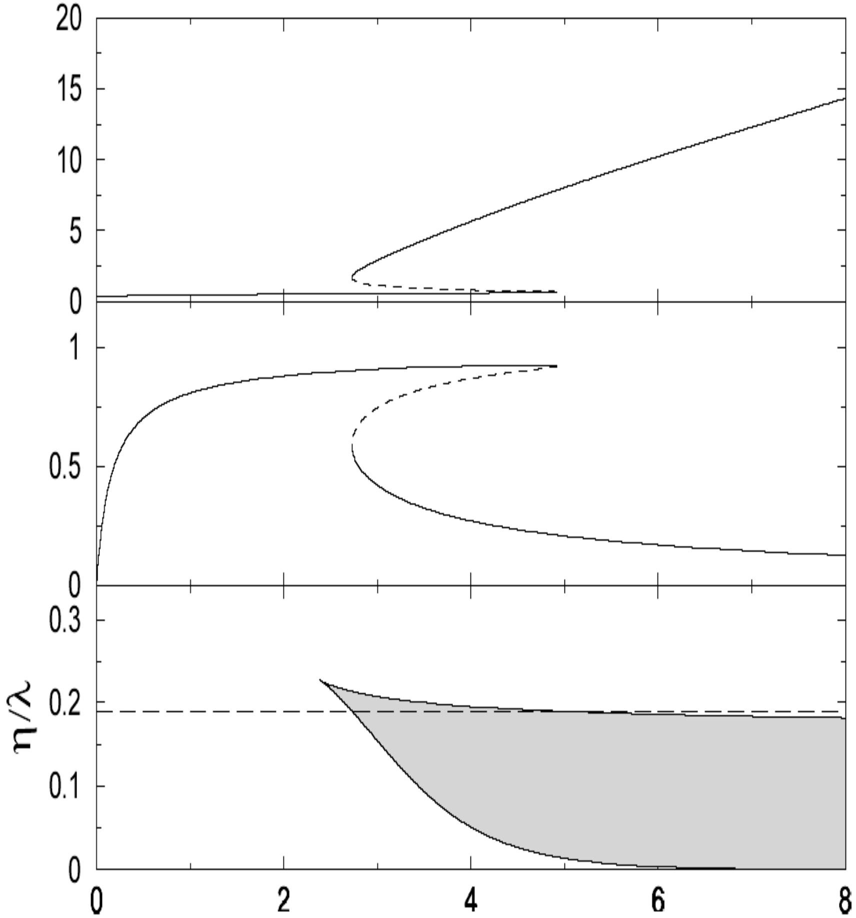


M.Marsili, F. Slanina & F. Vega-Redondo, Proc. Nat. Acad. Sci. **101**: 1439-1442 (2004)

Simulations: Mean-field theory:



ξ/λ = local search/volatility rates



ξ/λ = local search/volatility rates

- Sharp transition
- Coexistence/hysteresis
- Density of cliques (i.e. clustering) **decreases** with rate ξ at which they form in the dense phase!

M.Marsili, F. Slanina & F. Vega-Redondo, Proc. Nat. Acad. Sci. **101**: 1439-1442 (2004)

Conclusions

Under somewhat broad conditions

Sharp transitions: socio-economic networks are expected to emerge in an abrupt manner as a consequence of the feedback between networking efforts of individuals and the benefits the network provides in terms of coordination, information and innovation diffusion, social cohesion, …

Resilience: once dense networks form, they are robust to deterioration of external conditions

Coexistence: for the same environmental parameters, the network can either be dense or very sparse, depending on the history

Questions:

- When do we expect things to happen smoothly?
- Node volatility vs link volatility?
- A closer look into empirical data?
- "nucleation theory"?

References:
MM, F. Slanina, F.Vega-Redondo, Proc. Nat. Acad. Sci. 2004
G.C.M. Ehrhardt, MM, F.Vega-Redondo Phys. Rev.E 2006, Int. J.Game Theory 2006
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