Opinion dynamics: continuous and bit-strings opinion models

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Reminder on binary opinions

Binary opinions models (Ising spins) evolves through interactions towards large clusters of zero and ones (coarsening of random distribution). The details of which attractor is actually reached depends upon the details of the interactions, full mixing or network structures, reversible or irreversible opinion change, initial conditions, etc. All attractors are made up of combinations of 0 and 1 opinions, in variable proportions and eventual localisation.

This talk:

- What happens when a larger variety of opinions is actually available? Discussion of real and binary strings opinion dynamics.
- What role can have opinion makers? Co-evolution dynamics.

Continuous variables

Opinions represented by a **continuous variable x(t)** + **bounded confidence d**.

- Right/left political opinions.
- Opinion about evaluations, e.g. about economic utilities of possible choices.

Dynamics: random choice of two agents with opinion $x(t)$ and $x'(t)$; They actually interact only if $|x - x'| < d$ according to:

$$x(t+1) = x(t) + \mu \cdot (x'(t) - x(t))$$
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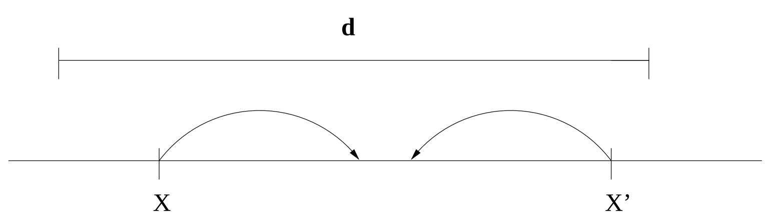


Threshold for interaction d: "openness" or rather uncertainty. μ is an intensity coefficient which mostly influences the speed of convergence towards the attractor.

Consensus attractor

Initial conditions: uniform distribution of opinions on the interval $[0, 1]$. Full mixing topology. For large threshold values, consensus is observed:

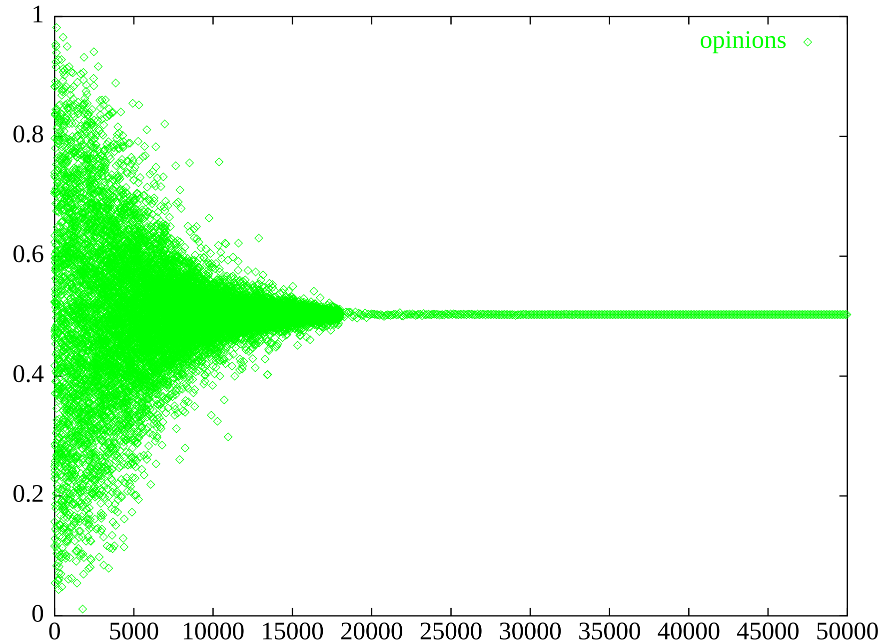


Abbildung 1: Time evolution of opinions for a threshold of 0.5.

When the threshold is larger than half the width of the distribution (or with no threshold), convergence towards average opinion is observed.

Clusters attractor

For lower threshold values, several clusters can be observed:

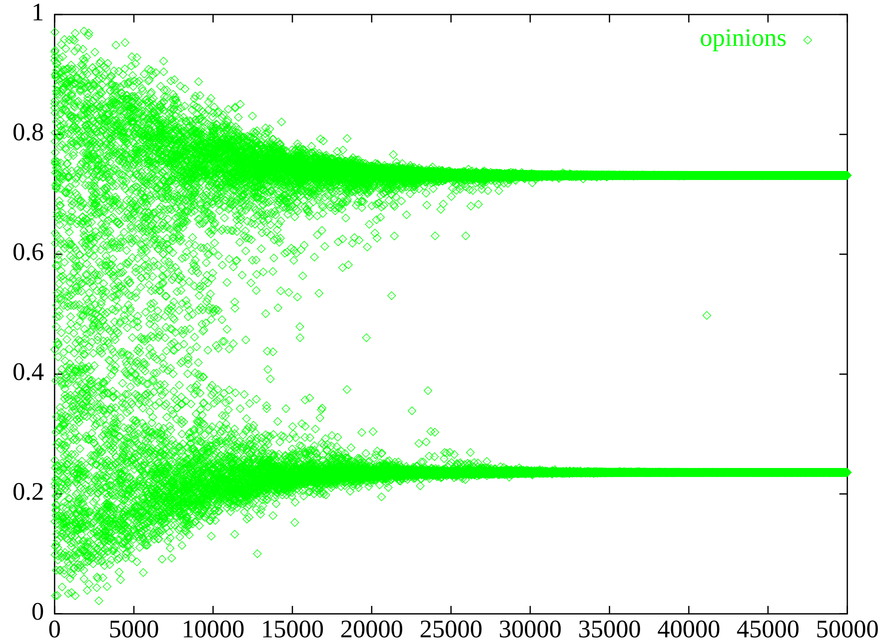


Abbildung 2: Time evolution of opinions for a threshold of 0.25 in a full-mixing topology.

Axelrod (1997): " If people tend to become more alike in their beliefs, attitudes and behavior when they interact, why do not all differences eventually disappear?"

The answer to Axelrod's paradox is the bounded confidence condition. For low confidence thresholds, some people get closer to others in the same cluster, but they don't interact anymore with people from different clusters, and their differences are actually increased.

Empirical background for continous opinions

Stylised facts rather than checking scaling.

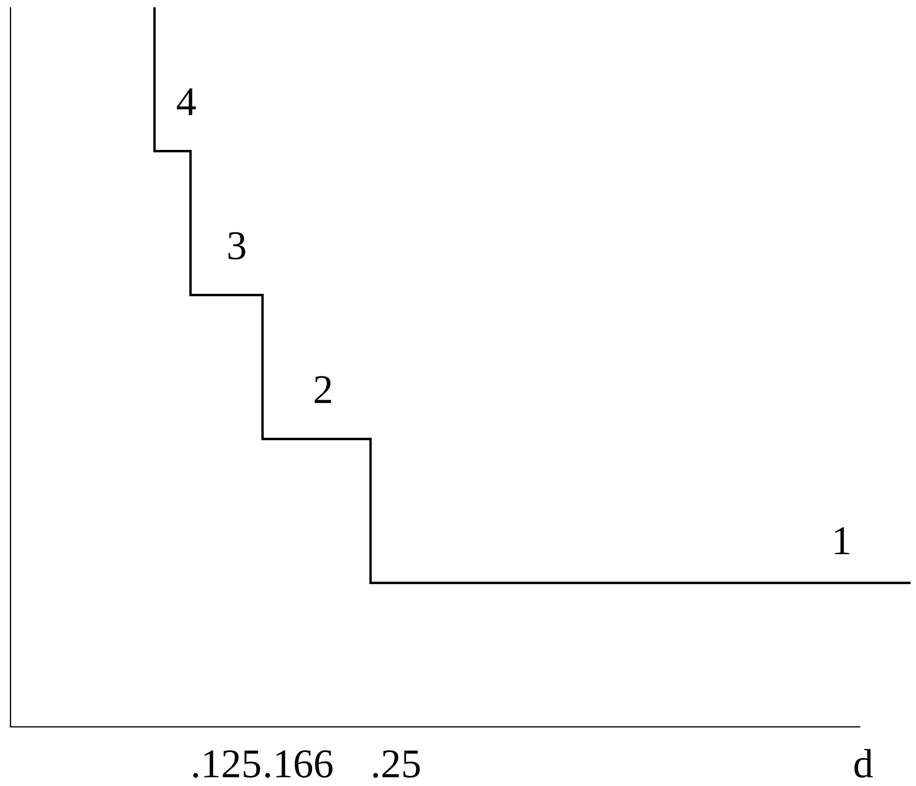
Henrich et al. compared through experiments shares accepted in the ultimatum game: people agree upon what a "fair" share should be, which can of course vary across different cultures.

Young and Burke report empirical data about crop sharing contracts, whereby a landlord leases his farm to a tenant laborer in return for a fixed share of the crops. In Illinois as well as in India, crop sharing distributions are strongly peaked upon "simple values" such as 1/2-1/2 or 1/3-2/3.

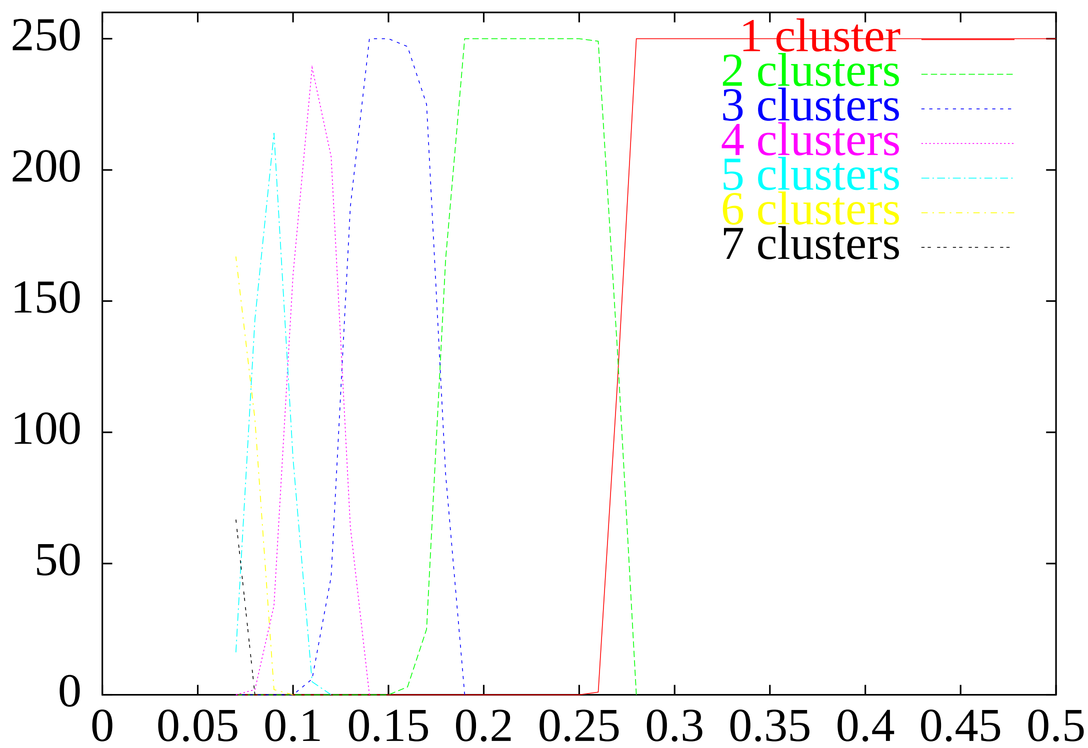
1992 New Agricultural Policy in EC. Decisions made by farmers about accepting or refusing grants to change their agricultural practices in favour of environmental-friendly practices; they first evaluate the economic outcome of new practices and compare them to their old practice in order to take their decision.

Scaling

Cluster number scales as the integer part of $1/2d$ (maximum number of clusters compatible with d).



The average number of clusters displays a descending staircase as a function of d.



Probabilities of observing 1, 2, 3...clusters as a function of uncertainty d.

More details in F. Vazquez,P. L. Krapivsky and S. Redner (2003) (2003) and E. Ben-Naim, P. L. Krapivsky, and S. Redner (2003). Master equations approach, minority clusters.

The network and lattice architectures display comparable results: clusters of contiguous uniform opinions, close to the positions obtained in the full mixing case. Weisbuch etal (2002), G. Weisbuch (2004).

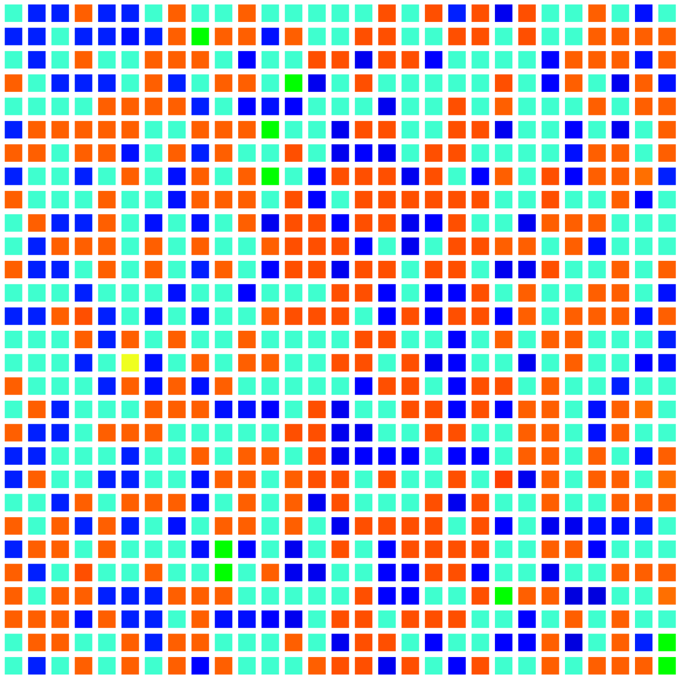


Abbildung 3: Time evolution of opinions for a threshold of 0.25 in a full-mixing topology.

Bit strings dynamics

Let us think of the cultural or political domains. A culture or a political opinion can be described by several features. Axelrod model of the diffusion of cultures. The "meme" approach to cultural studies.

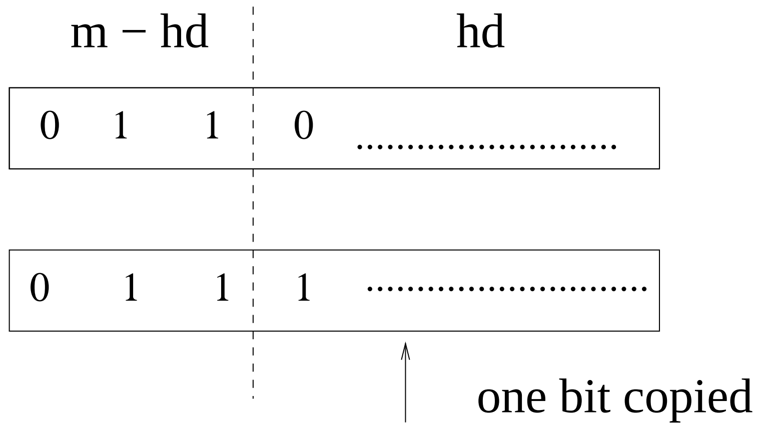
In politics, think of different issues such as European integration, public pension system, social security, abortion, drug consumption etc. A political opinion can be defined by a string of binary choices (for or against).

Each voter then possess a political opinion modeled by a bit string, with m bits, where each bit codes for her opinion about one feature. Voters can be influenced by others on the occasion of encounters.

($m = F_{Axelrod}, q_{Axelrod} = 2$, full-mixing topology rather than lattice).

Bit strings dynamics 2

Dynamics of opinion exchanges:



- Random sampling of a pair:
- $iff\ h_d \leq d$:

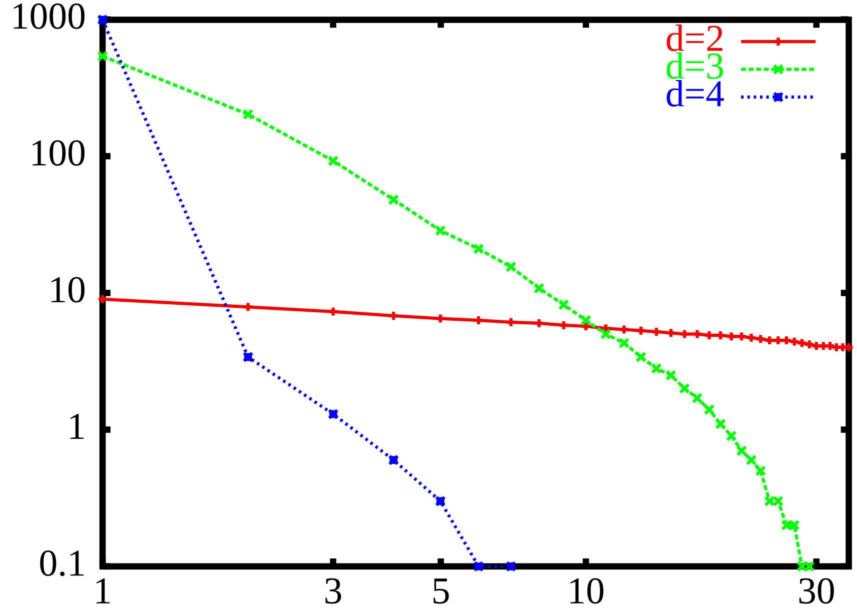
h_d is the Hamming distance between the two opinion strings (# of different bits). At each time step, two agents with different opinion are randomly selected (full mixing hypothesis). Agents only interact if their Hamming distance is less than the threshold d. One agent, randomly chosen, then takes randomly the opinion of the other one on one single topic.

What are the attractors of the dynamics?

- Two opposite opinions as in the case of ferro-magnetism ?
- Consensus?
- Many clusters?

Compare with sexual reproduction and Evolution of species.

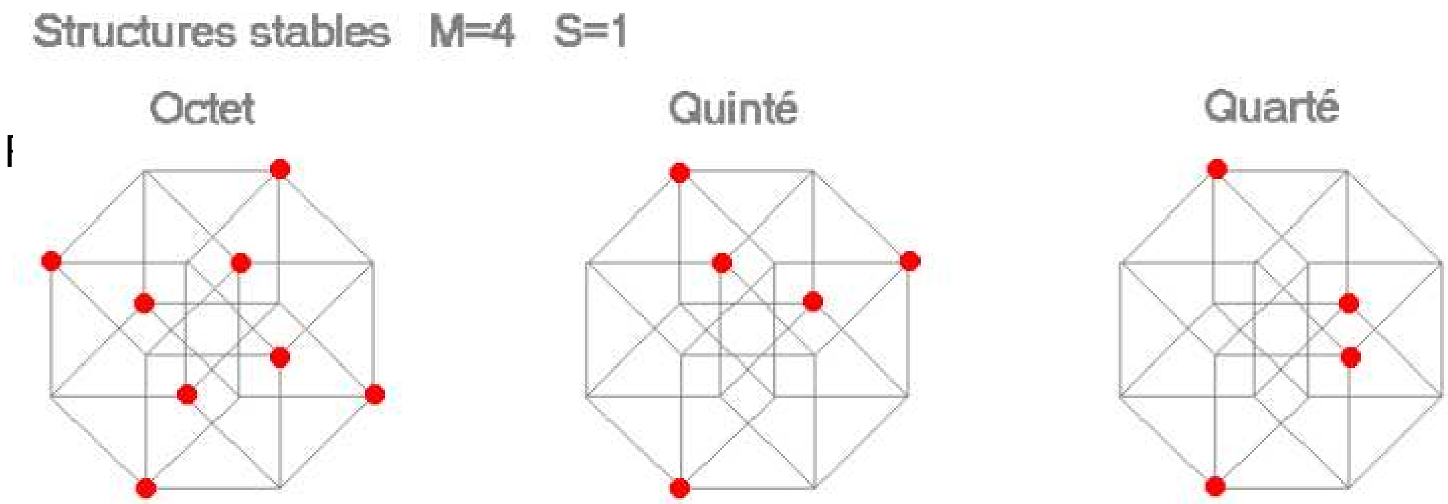
Bit strings dynamics



Zipf plot: Attractors sizes arranged by decreasing order (13 bit strings, 1000 individuals, d, interaction threshold (max number of different bits for interaction), each point is an average over 200 random samples.

For $d \geq 4$, consensus.

For $d = 3$, Zipf distribution ??



Attractor configurations for $m = 4$, $d = 2$

Transition from consensus to an exponential (in m) number of clusters around $d = 3$ threshold.

Co-evolution

Politicians, State agencies, Producers etc. are interested in voters, constituency, consumers etc. opinions to increase the power (number of votes, policy enforcement, profits and market share).

How should they position their offer in view of the opinion dynamics of their "customers"?

Models of co-evolution of producers and consumers choices.

Reminder on binary choice models, with adjustable economic utility by one single producer:

Minimal producer rationality. Producer reaction to market share evolution results in SOC. Solomon S., Weisbuch G., de Arcangelis L. , Jan N., and Stauffer D. (2000).

Full producer rationality: knowing the attractor of the dynamics of consumers' opinion the producer chooses the product utility which maximises its profit. First order transitions in prices and market shares. Mirta B. Gordon, Jean-Pierre Nadal, Denis Phan and Jean Vannimenus (2005), Jean-Pierre Nadal, Denis Phan, Mirta B. Gordon and Jean Vannimenus (2005). (based on IWTP, Sandro Fortunato, J.P. Bouchaud on babies and cell-phones).

Both predictions quite different from General Equilibrium Theory !!

Co-evolution on the hypercube

When the political options (resp. the products characteristics) are multiple, the political parties (resp. the producers) offer political platforms (resp. products) enabling them to optimize votes in their favour (resp. profits).

Lets us discuss specifically producers and consumers dynamics.

T. Araujo, R. Vilela Mendes: Market-oriented innovation: When is it profitable? An abstract agent-based study,WP31/2006/DE/UECE,(2006).

"Hiking the hypercube: producers and consumers", Tanya Araujo, G. Weisbuch (2007)

In Biology similar models apply to immunology (Ab/Ag recognition) or Co-evolution of species (Red Queen dynamics). The bit strings then represent genes or sets of phenotypic traits.

Genome =) recognition =) action =) results =) population change

Bit string model of producers and consumers.

Set of interacting producers and consumers.

Each producer j produces a product defined by a bit-string S_j of length m.

Every consumer i has a set of binary preferences S_i of length m for the features of the product.

Transactions are based on the overlap between product features and consumer preferences:

$$q_{ij} = \sum_{l=1}^{m} S_i^l.EQ.S_j^l \tag{1}$$

A consumer chooses the product which best overlaps its preferences and a transition occurs if the overlap is larger than the threshold θ.

Genome =)$recognition$ =) action =) results =) population change

Updating satisfaction and capital

In the course of time (parallel iteration) cumulative quantities applying to consumers and producers are updated according to:

$$S_i(t+1) = S_i(t) - a_c + \frac{q_{ij}}{m} \quad (2)$$

Satisfaction is increased according the relative overlap of the need and product strings. It is decreased by a constant cost per transaction a_c.

$$C_j(t+1) = C_j(t) - a_p + \sum_{i(j)} \frac{q_{ij}}{m} \quad (3)$$

Capital is increased by the set of transactions in which producer j was involved, according the relative overlaps. The index $i(j)$ runs over all the consumers i that were supplied by producer j. Capital is decreased by a constant production cost a_p.

Whenever S_i becomes negative, the consumer is replaced by a new consumer with random preferences. Producers with negative capital simply disappear.

Genome =) recognition =)*action =) results =) population change*

Model 0: passive producers.(T.A. and G.W.)

Consumers look for producers with fixed product characteristics.

Pure selection dynamics: consumers which are able to find producers satisfying their needs survive provided that:

$$a_c \geq \frac{q_{ij}}{m} \tag{4}$$

$a_c = 1$ is critical between two possible regimes for consumers: below 1, a constant random renewal of consumers occurs. Above 1, some consumers with adequate preferences are selected, survive and their satisfaction increases.

A similar selection of producers is observed depending upon the values of a_p, N_c and N_p.

Time plots.

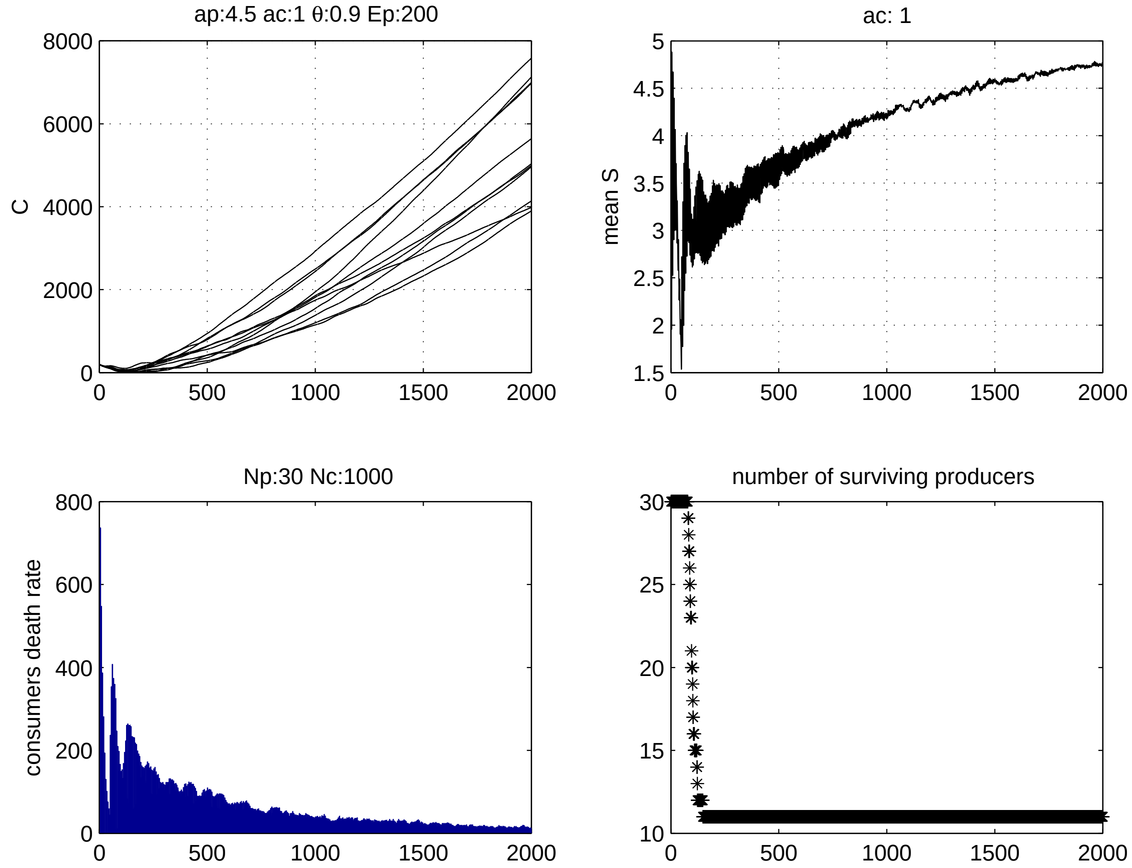


Abbildung 4: Time evolution of producers capital C_i, of the average consumer satisfaction S, of the number of consumer death per time unit and the number of surviving producers. N_p initial was 30, $N_c = 1000$, $a_c = 1$, $C_p = 200$, $\theta = 9$.

Niches vs Competition regimes.

The threshold for interaction θ fixes a basin of satisfaction in the neighborhood of the producer, which size is given by the number of sites $n_s(\theta)$ which Hamming distance to its product string is less or equal to $m - \theta$.

Any consumer which string is in the basin of satisfaction of a producer may be involved in transactions with her. For $m = 10$, $\theta = 10$ gives a neighbourhood of size 1, size 11 for $\theta = 9$, size 56 for $\theta = 8$ etc.

Comparing $n_s(\theta) \times N_p$ to $2^m = 1024$, the number of sites on the hypercube, gives an approximation of the condition in threshold separating the competition regime at low θ values, from the "niche" regime.

In the competition regime, customers may choose between different producers in their neighborhood; producers then compete for customers.

In the niche regime, they are so far apart that they don't have common customers and they don't compete.

Very different conclusions from the two ice cream sellers on the beach (1d, Hoteling)!

The evolving producer (T.A. and R.V.M.)

Compare the performance of passive producers with those of an active producer. Products of passive producers have fixed characteristics.

Active producer strategy: bounded rationality. Evolves bit per bit towards the center of gravity of her consumer preferences (equivalent to hill climbing).

In the niche regime, no advantage to the active producer was observed.

In the competition regime, for $\theta = 0.6$ and $a_c = 2$:

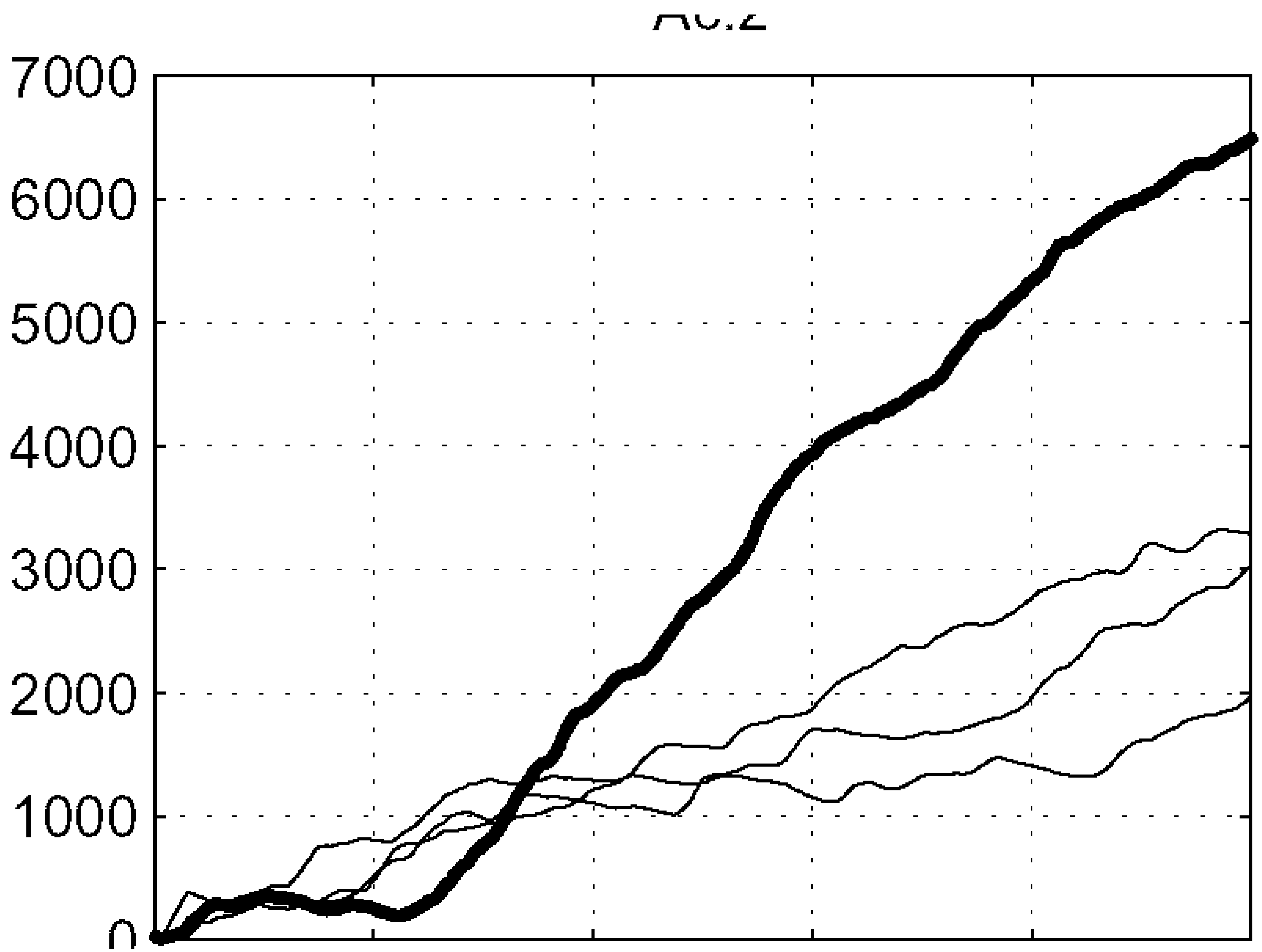


Abbildung 5: Time evolution of producers capital. Thicker line: evolving producer capital.

Conclusions

For continuous and bit-strings opinions:

Bounded confidence brings clustering. The number of clusters depends upon the threshold for interaction. Dimension of the state space matters !!

Scalar opinions

Starting from lower thresholds, staircase decrease of the number of clusters. $Int(1/2d)$ rule.

Bit strings

Abrupt change from an exponential number of clusters to consensus.

Co-evolution

A largely unexplored frontier. Many possible applications including some very practical, e.g. the car dealer problem.

Thanks

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